DIFFERENTIAL MODULATION (PART II: MIMO SYSTEM)

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PLAN

- Some preliminary about Gaussian distribution
- Coherent MIMO transmission
- Non Coherent MIMO transmission

GAUSSIAN DISTRIBUTION

- $X \sim N(0, \sigma^2) \implies p_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right)$
- $X_1, X_2 \sim N(0, \sigma^2)$ *and iid* $\implies$

$$p_{X_1 X_2}(x_1, x_2) = \frac{1}{(2\pi)\sigma^2} \exp\left(-\frac{x_1^2 + x_2^2}{2\sigma^2}\right)$$

- $X \sim CN(0, \sigma^2) \implies P_X(X) = \frac{1}{\pi\sigma^2} \exp\left(-\frac{|X|^2}{\sigma^2}\right)$
- $X_1, X_2 \sim CN(0, \sigma^2) \implies$

$$P_{X_1, X_2}(X_1, X_2) = \frac{1}{\pi^2\sigma^4} \exp\left(-\frac{|X_1|^2 + |X_2|^2}{\sigma^2}\right)$$

- X a complex column vector of size n.

$$P_X(X) = \frac{1}{\pi^n \sigma^{2n}} \exp\left(-\frac{\sum_{i=0}^{n} |X_i|^2}{\sigma^2}\right) = \frac{1}{\pi^n \sigma^{2n}} \exp\left(-\frac{X^H X}{\sigma^2}\right)$$

GAUSSIAN DISTRIBUTION (CONT.)

- X a complex column vector of size n correlated.

$$P_X(X) = \frac{1}{\pi^n \det(\Sigma)} \exp(-X^H \Sigma^{-1} X)$$

- X a complex matrix of size $n \times m$ (independent).

$$P_X(X) = \frac{1}{\pi^{nm} \sigma^{2nm}} \exp\left(-\frac{Tr\{XX^H\}}{\sigma^2}\right)$$

- X a complex matrix of size $n \times n$ correlated.
  - It can be changed to a column vector of size n^2 and the previous formulas for a vector holds.
  - Or directly in matrix form (next slide)

GAUSSIAN DISTRIBUTION (CONT.)

- Define: $\Sigma_X = E\{XX^H\}$ $and\,\Omega_x = E\{X^H X\}$

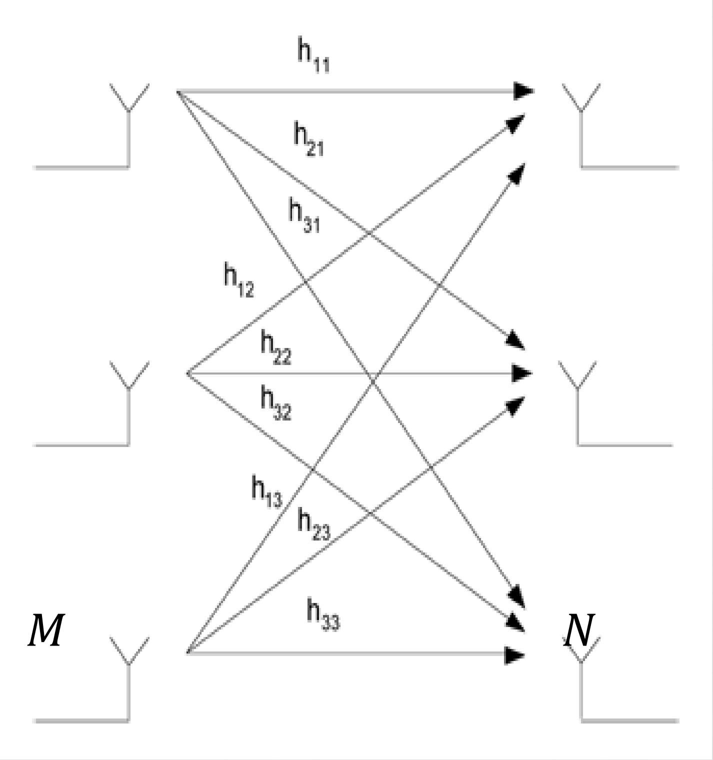
$$P_X(X) = \frac{1}{\pi^{n\times m}|\Sigma_X|^m|\Omega_X|^n}\exp(-Tr\{\Sigma_X^{-1}X\Omega_X^{-1}X^H\})$$

- If $\Sigma_X = \alpha I$ and $\Omega_X = \beta I$, then

$$P_X(X) = \frac{1}{\pi^{nm}\alpha^{nm}\beta^{mn}}\exp\left(-Tr\left\{\frac{XX^H}{\alpha\beta}\right\}\right)$$

SYSTEM MODEL

- MIMO with M transmit and N receive antennas.
- Rayleigh fading channel (i.i.d.)
- At time t TX sends the complex symbols x_{tm} on TX antennas $m = 1,2,\ldots M$.
- At RX, we receive y_{tn} for $n = 1,2,\ldots,N$.
- $y_{tn} = \sqrt{\rho}\sum_{m=1}^{M} h_{mn}x_{tm} + w_{tn}$, $t = 0,1,\ldots$ and $n = 1,2,\ldots,N$



SYSTEM MODEL

- The fading coefficients are assumed to be i.i.d. in n and m. $h_{nm} \sim CN(0,1)$.
- The noise is i.i.d. for m and t: $w_{tm} \sim CN(0,1)$.
- The transmitted symbols are normalized such that

$$E \sum_{m=1}^{M} |x_{tm}|^2 = 1$$

- With these definitions, ρ is the expected SNR at **each receive antenna** and the total transmitted power is independent of M.

TRANSMITTING SIGNAL CONSTELLATION

- Suppose a spectral efficiency of ν bits per channel use is required.
- Since there are T channel uses per block, we need νT bits to select an $M \times T$ matrix.
- Therefore we need $2^{\nu T}$ $M \times T$ different matrices, which is the symbol constellation set.
- Each entry of these matrices can be an MPSK symbol.

RECEIVED SIGNAL

- A block of T consecutive channel uses forms one transmission.
- We assume that the channel does not change significantly during $2T$ channel uses.
- The received matrix can be written

$$Y_{N\times T} = \sqrt{\rho}\,\mathrm{H}_{\mathrm{N\times M}} X_{\mathrm{M}\times T} + \mathrm{W}_{\mathrm{N}\times T}$$

COHERENT DETECTION

$$Y_{N\times T} = \sqrt{\rho}\,\mathrm{H}_{\mathrm{N\times M}} X_{\mathrm{M}\times T} + \mathrm{W}_{\mathrm{N}\times T}$$

- The channel is known at RX
- Because everything is decorrelated

$$p(Y|H,X) = \frac{1}{\pi^{NT}} \exp\left(-Tr\{(Y-\sqrt{\rho}HX)(Y-\sqrt{\rho}HX)^{\mathrm{H}}\}\right)$$

- The ML receiver

$$\hat{S}_t = \arg\max_m p(Y|H,X_m) =$$
$$\arg\min_m Tr\{(Y-\sqrt{\rho}HX_m)(Y-\sqrt{\rho}HX_m)^{\mathrm{H}}\}$$

PAIR WISE ERROR PROBABILITY

- Chernoff bound

$$p\{X_0 \to X_1\} \le \frac{1}{\left|I + \frac{\rho}{4}(X_0 - X_1)(X_0 - X_1)^H\right|^N}$$

$$= \frac{1}{\prod_{n=1}^{r}\left(1 + \frac{\rho\lambda_n}{4}\right)^N}$$

- For large SNR ($\rho \to \infty$):

$$p\{X_0 \to X_1\} \le \frac{1}{\left(\frac{\rho}{4}\right)^{NM} |(X_0 - X_1)(X_0 - X_1)^H|^N}$$

$$= \prod_{n=1}^{r}\left(\frac{\rho\lambda_n}{4}\right)^N$$

- Diversity order = NM (if rank is less than M the diversity will be rM.
- Coding gain $\prod_{n=1}^{N}\lambda_n = \ det\{(X_i - X_j)(X_i - X_j)^H\}$

- Conclusion: for a good code, the above matrix is full rank for all i and j; and the min of the above determinant for all I and j is maximum.

NON COHERENT DETECTION

- Therefore we need $2^{\nu T}$ elements in the matrix group $\mathcal{G}$.
- Each matrix is supposed to be unitary: $S_i \in \mathcal{G}$, $S_i S_i^H = S_i^H S_i = I$.
- This modulation is called Unitary Space-time modulation

$$X_t = X_{t-1} S_t$$

- X_0 is a fixed unitary matrix with no information.

Note: We assume that $T = M$.

ML RECEIVER

$$Y_{t-1} = \sqrt{\rho} H_{t-1} X_{t-1} + W_{t-1}$$
$$Y_t = \sqrt{\rho} H_t X_t + W_t = \sqrt{\rho} H_t (X_{t-1} S_t) + W_t$$
$$Y_t \approx Y_{t-1} S_t - W_{t-1} S_t + W_t$$
$$Y_t \approx Y_{t-1} S_t + Z_t$$

- Because S_t is a unitary matrix, the $W_{t-1} S_t$ has the same statistics as W_t. Therefore $E\{Z_t^H Z_t\} = 2I_M$.

- The ML receiver calculates the following

$$\hat{S}_t = \arg\min_{S_k} \|Y_t - Y_{t-1} S_k\|^2$$

Simplifies: $\hat{S}_t = \arg\max_{S_k} Re\{Tr(Y_{t-1} S_k Y_t^H)\}$

PAIR WISE ERROR PROBABILITY

$$Y_t \approx Y_{t-1} S_t + Z_t$$
$$Y_{t-1} = \sqrt{\rho} H_{t-1} X_{t-1} + W_{t-1}$$

- We try to calculate the statistics of Y_{t-1}.
- Y_{t-1} is a complex Gaussian random matrix.

- $\Sigma_{Y_{t-1}} = E\{Y_{t-1} Y_{t-1}^H\}$ for simplicity : $\Sigma_{Y_{t-1}} = E\{Y Y^H\}$

$$\Sigma_{Y_{t-1}} = E\{(\sqrt{\rho} HX + W)(\sqrt{\rho} HX + W)^{\mathrm{H}}\}$$
$$= \rho E\{HXX^H H^H + WW^H\}$$
$$= \rho E\left\{HX_0 S_k S_k^H X_0^H H^H + WW^H\right\}$$
$$= \rho I + I$$

PAIR WISE ERROR PROBABILITY

- $Y_t \approx Y_{t-1}S_t + Z_t = \sqrt{1+\rho}\widetilde{H}S_t + Z_t$
- $Y_t' = \frac{Y_t}{2} = \sqrt{\frac{1+\rho}{2}}\widetilde{H}S_t + Z_t'$
- Comparing with that of coherent detection and for large SNR
- $p\{S_0 \rightarrow S_1\} \leq \frac{1}{\left(\frac{1+\rho}{2\times 4}\right)^{NM}\left|(S_0-S_1)(S_0-S_1)^H\right|^N}$
- For large SNR and compare to coherent case, 3 dB of degradation observed.

TO BE CONTINUED