# DIFFERENTIAL MODULATION (PART I: SYSTEM SISO)

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# CHANNEL MODEL

Clark's channel model

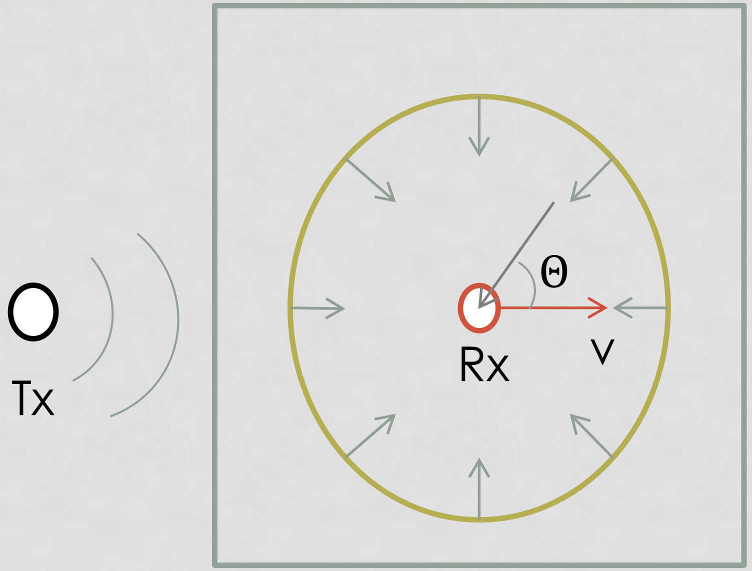
Suppose

- Uniform power distribution
- Isotropic Rx Antenna
- High scattering environment
- Narrow band transmission: $w$ is much smaller than the reciprocal of the delay spread → single tap transmission (flat fading)

$$y(t) = \int_0^{2\pi} a_\theta x(t - \tau_\theta) d\theta$$

$y(t)$ received signal

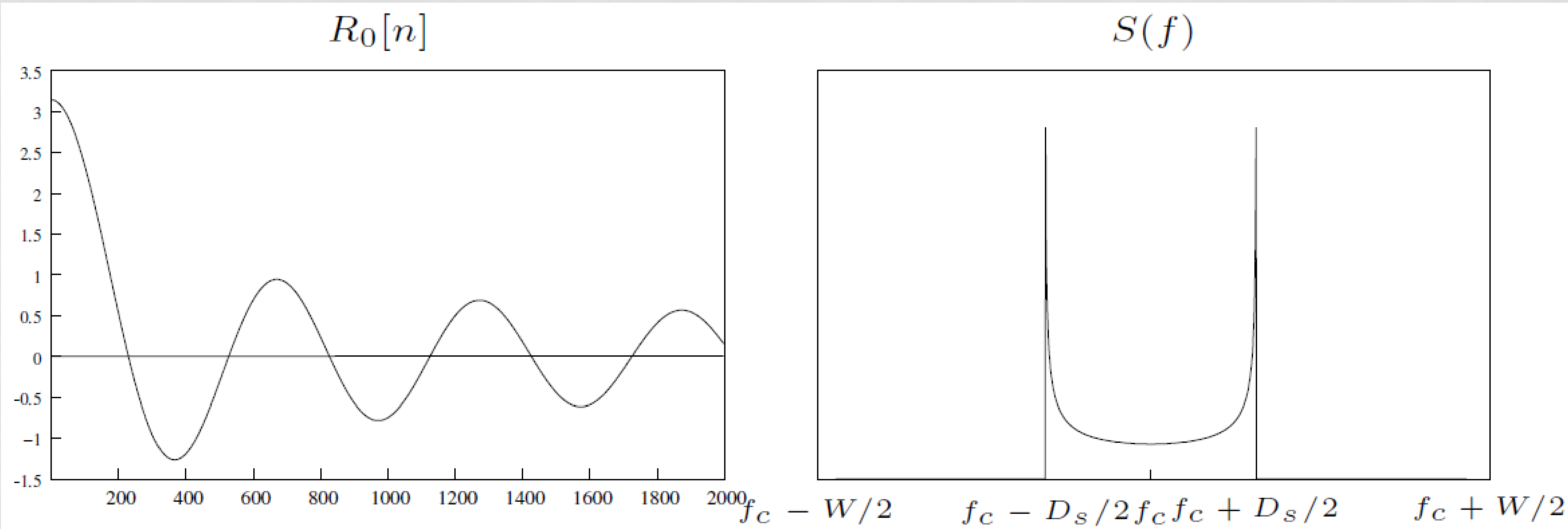
$$y[m] = h_0[m]x[m] + w[m]$$



# CLARK'S MODEL

- The phase of signal arriving from the direction $\theta$ and at time 0 is: $2\pi f_c \tau_\theta(0)$.
- $\theta \sim U(0,2\pi)$.
- The process $h_0[m]$ is stationary, Gaussian with the autocorrelation function: $R_0[nT_b] = J_0(2\pi f_d n T_b)$
- $J_0(.)$ the 0th-order Bessel function of the first kind: $J_0(x) = \frac{1}{\pi}\int_0^{\pi} e^{-jx\cos\theta} d\theta$ and $f_d = f_c v/c$ and $D_s = 2f_d$ is the Doppler spread.
- The PSD of $h_0[m]$ is $S(f) = \frac{1}{\pi f_d \sqrt{1-\left(\frac{f}{f_d}\right)^2}}$ for $-f_d \leq f \leq f_d$

# AUTOCORRELATION FUCNTION AND PSD




If we define the coherence time $T_c = n\frac{1}{w}$ and it is where $R_0[n] = 0$, then

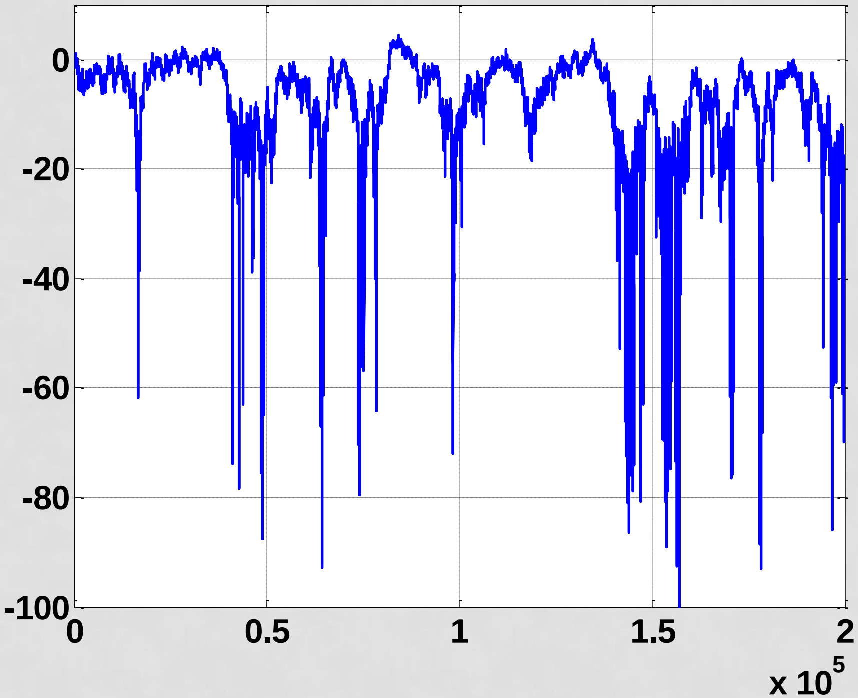
$T_c = \frac{j_0^{-1}(0)}{2\pi f_d} = \frac{0.3828}{f_d} = 0.3828\frac{c}{v f_c}$

Example: $f_c = 2$ GHz, $v = 90$km/h → $T_c = 2.3$ ms

# SIMPLE CHANNEL MODEL

- In simulation some times the following model is used,

- Channel is modeled by a first order autoregressive process:
  $h_k = \alpha h_{k-1} + \sqrt{1-\alpha^2} n_k$
  where
  $\alpha = E\{h_k h_{k-1}\} = J_0(2\pi f_d T_b)$.



# SISO DIFFERENTIAL COMMUNICATIONS

- Supposing M-PSK modulation scheme
- Spectral efficiency: $\log_2 M$ bit/s/Hz
- Sequence to be transmitted $c_1 c_2 c_3 \ldots \mathrm{c_t} \ldots$ where $c_t \in \{0,1,\ldots,M-1\}$.
- The modulated sequence $s_1 s_2 s_3 \ldots s_t \ldots$ with $s_t = e^{j2\pi c_t/M}$.
- The transmitted sequence is $x_0 x_1 x_2 \ldots x_t \ldots$. Where $x_0$ can be any point of constellation for example $x_0 = 1$.

$$x_t = s_t x_{t-1} = x_{t-1} e^{j2\pi c_t/M}.$$

Note, there is no constellation expansion.

# DECISION RULE

- The transmitted sequence is $x_0 x_1 x_2 \ldots x_t \ldots.$ Where $x_0$ can be any point of constellation for example $x_0 = 1$.

$$x_t = s_t x_{t-1} = x_{t-1} e^{j2\pi c_t / M}$$

- Received sequence: $r_0\, r_1 r_2 \ldots r_t \ldots$

- The receiver calculates $\hat{\theta}_t = \arg r_{t-1}^{*} r_t$.

# ANALYSES FOR BPSK

- $y_t = h_t x_t + w_t = h_t x_{t-1} s_t + w_t$
- $y_{t-1} = h_{t-1} x_{t-1} + w_{t-1}$
- Therefore $y_t \approx y_{t-1} s_t - w_{t-1} s_t + w_t = y_{t-1} s_t + z_t$
- $z_t$ remains Gaussian with $\sigma_z^2 = 2\sigma^2$
- The random variable $y_{t-1}$ is also zero mean Gaussian with variance $1 + \sigma^2$

# BER PERFORMANCE

- The decision metric is $r = Re\left\{\frac{y_{t-1}^{*}}{|y_{t-1}|} y_t\right\} = |y_{t-1}| \mathrm{s_t} + \mathrm{z_t'}$ where $z_t'$ is a real random variable with $z_t' \sim N(0, N_0)$
- If the signal $s_t = \pm 1$, the BER given $y_{t-1}$ is
$$Q\left(\frac{|y_{t-1}|}{\sqrt{N_0}}\right)$$
- The BER is the expected value of $Q\left(\frac{|y_{t-1}|}{\sqrt{N_0}}\right)$ over the distribution of $|y_{t-1}|$.

# HIGH SNR PERFORMANCE

- For high SNR, $N_0 \to 0$ and the distribution of $|y_{t-1}|$ is the same as $|h_t|$ therefor an ordinary Rayleigh channel, however the power of noise is doubled.
- $p_e|h \approx Q\left(\frac{|h|}{\sqrt{N_0}}\right) = Q\left(\sqrt{|h|^2\bar{\gamma}}\right)$ where $\bar{\gamma}$ is the mean of received SNR: $\bar{\gamma} = E\left\{\frac{|h|^2}{N_0}\right\} = 1/N_0$.
- For high SNR: $p_e = E\left\{Q\left(\sqrt{|h|^2\bar{\gamma}}\right)\right\} = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{2+\bar{\gamma}}}\right) \approx \frac{1}{2\bar{\gamma}}$
- It means that the differential modulation is 3 dB worse than coherent modulation (for coherent we had $p_e \approx \frac{1}{4\bar{\gamma}}$).

# VARYING CHANNEL

- The channel is modeled as: $h_t = \alpha h_{t-1} + \sqrt{1-\alpha^2} n_t$ where
$$\alpha = E\{h_t h_{t-1}\} = J_0(2\pi f_d T_b)$$
- The received signal:
- $y_t = x_t h_t + w_t = x_{t-1} s_t h_t + w_t$
- $y_t = \ x_{t-1} s_t \left[\alpha h_{t-1} + \sqrt{1-\alpha^2} n_t\right] + w_t$
- $y_t = \alpha y_{t-1} s_t - \alpha s_t w_{t-1} + \sqrt{1-\alpha^2} s_t x_{t-1} n_t + w_t$
- $y_t = \alpha y_{t-1} s_t + z_t + \sqrt{1-\alpha^2} \mathrm{n}'_{\mathrm{t}}$
- $z_t \sim N(0, 2N_0)$, the second one is the estimation error and cannot reduced with SNR => error floor

# CONCLUSION

- Differential modulation is very appreciated where there are too much channels to be estimated
- However, a 3 dB degradation is observed in the original version in comparison with a coherent scheme
- In time varying channel an error floor effect is observed that can be removed by multiple symbol detection.