MIMO systems, fast fading channels and differential detection

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Motivation

- Fading is fast in comparison with block duration
- Time diversity exists
- Channel varies continuously
    - Channel estimation is difficult
    - Present estimation is not good for the future symbols
- Differential techniques should be appreciated

Conventional Techniques

Using differential modulation in transmitter

$$\mathbf{X}_k = \mathbf{S}_{l(k)} \mathbf{X}_{k-1}$$

where $\mathbf{S}_i \in$ *generation constellation* and $\mathbf{X}_i \in$ *transmission constellation*

$$\mathbf{r}_{k-1} = \sqrt{E_s} \mathbf{X}_{k-1} \mathbf{h}_{k-1} + \mathbf{v}_{k-1}$$

$$\mathbf{r}_k = \sqrt{E_s} \mathbf{X}_k \mathbf{h}_k + \mathbf{v}_k$$

$$\mathbf{r}_k = \sqrt{E_s} \mathbf{S}_{l(k)} \mathbf{X}_{k-1} \mathbf{h}_k + \mathbf{v}_k$$

Channel is modeled by first order autoregressive process (it is assumed constant for an entire transmission matrix) :

$$\mathbf{h}_k = \alpha \mathbf{h}_{k-1} + \sqrt{1-\alpha^2} \mathbf{n}_k \text{ and } \alpha = \mathbf{E}\{h_k h_{k-1}\} = J_0(2\pi f_D T_B)$$

Analytical PEP

$$\mathbf{r}_k = \alpha \mathbf{r}_{k-1}\mathbf{S}_{l(k)} + \left\{\mathbf{v}_k - \alpha \mathbf{S}_{l(k)}\mathbf{v}_{k-1} + \sqrt{E_s}(1-\alpha^2)\mathbf{S}_{l(k)}\mathbf{X}_{k-1}\mathbf{n}_k\right\}$$

- If SNR $\rightarrow \infty, \mathbf{v}_k \rightarrow 0$ and $\mathbf{v}_{k-1} \rightarrow 0$ but $\sqrt{E_s}(1-\alpha^2)\mathbf{S}_{l(k)}\mathbf{X}_{k-1}\mathbf{n}_k$ does not vanish $\Rightarrow$ error floor
- There is analytical calculation for PEP ($\mathbf{S}_l$ unitary *i.e.* $S_i S_j^H = I_M$) [1]. The following limiting cases can result:

- Case 1: Quasi static channel $\alpha = 1$ (same as Chernoff bound)
- Case 2: High SNR $\frac{E_s}{\sigma_v^2} \rightarrow \infty$ (error floor)
- Case 3: Where $\mathbf{E}_{i,j}^H \mathbf{E}_{i,j} = \beta_{i,j}\mathbf{I}$ (like orthogonal designs)

Quasi static fading channel [1]

Here $\alpha = 1$, il means that $f_D T_B = 0$

$$\mathbf{P}(\mathbf{S}_i \rightarrow \mathbf{S}_j) \leqslant \frac{1}{2} \prod_{l=1}^{L} \left(\frac{1}{1 + \frac{E_s^2}{4\sigma_v^2 (2E_s + \sigma_v^2)} \lambda_l} \right)^{\mu_l}$$

where L is the number of distinct nonzero eigenvalues λ_l of $\mathbf{E}_{i,j}^H \mathbf{E}_{i,j}$ with multiplicity μ_l and $\mathbf{E}_{i,j} = \mathbf{S}_i - \mathbf{S}_j$
In the paper it is generalized to N receive Antennas.

Design Criterion [2]

- The performance depends on the properties of generator constellation. It is measured by "diversity product":

$$\zeta = \frac{1}{2} \min_{0 \leq l, l' < L} |det(\mathbf{S}_l - \mathbf{S}_{l'})|^{1/M}$$

- $0 < \zeta < 1$ and the closer to 1, the better the error performance
- Diagonal design is proposed
  - Simple coding, simple decoding
  - Gives full diverse code
  - No multiplexing gain

PEP for high SNR (floor effect) [1]

$$\mathbf{P}(\mathbf{S}_i \rightarrow \mathbf{S}_j) =$$

$$\frac{1}{2}\left\{1 - \sum_{l=1}^{L}\sqrt{\frac{\alpha^2\lambda_l}{4(1-\alpha^2)+\alpha^2\lambda_l}}\sum_{p=1}^{\mu_l} c_{p,l}\sum_{q=0}^{p-1}\left(\begin{array}{c} 2q \\ q \end{array}\right)\left\{\frac{1-\alpha^2}{4(1-\alpha^2)+\alpha^2\lambda_l}\right\}^q\right\}$$

where L is the number of distinct nonzero eigenvalues λ_l of $\mathbf{E}_{i,j}^H\mathbf{E}_{i,j}$ with multiplicity μ_l and $\mathbf{E}_{i,j} = \mathbf{S}_i - \mathbf{S}_j$

Note: The code is unitary $\mathbf{S}_l^H\mathbf{S}_l = \mathbf{I}$ and the question is how to design the code to minimize the PEP.

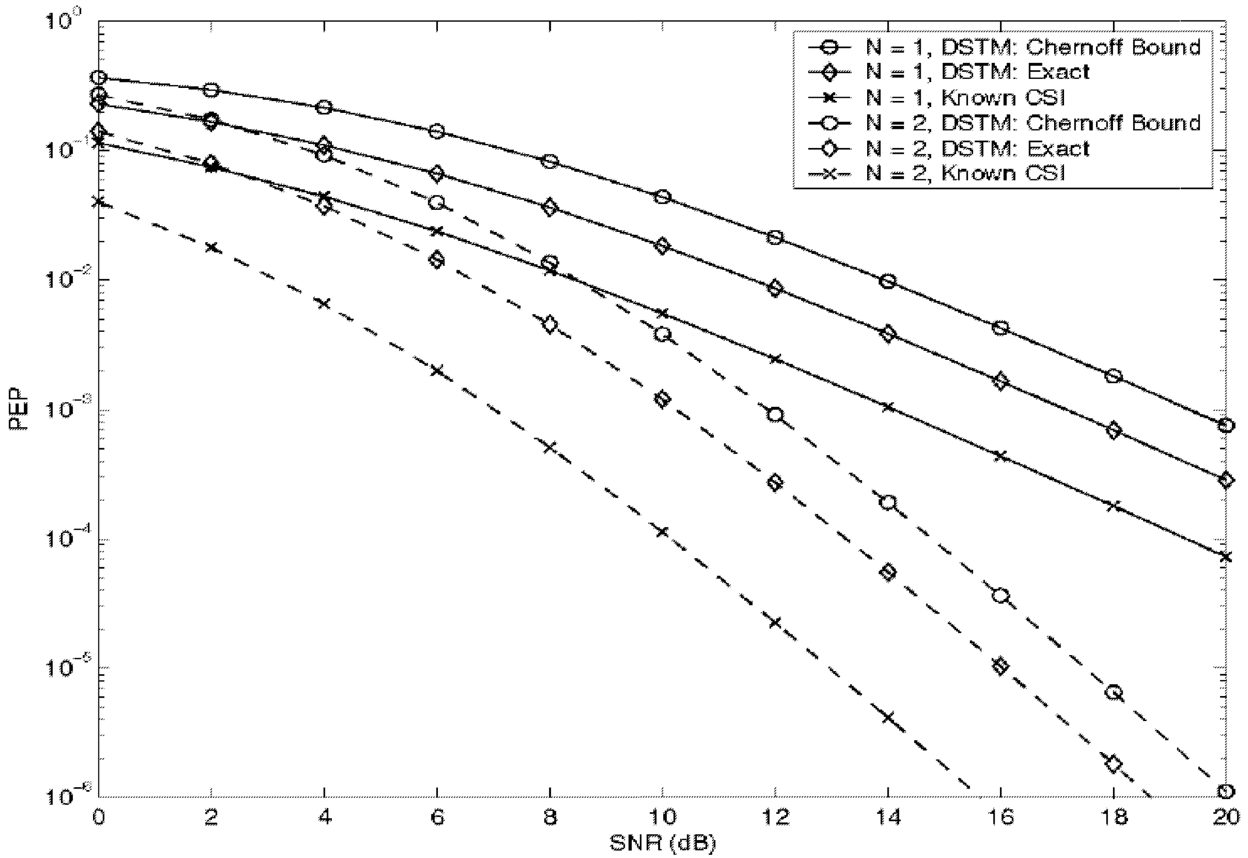
Special case $\mathbf{E}_{i,j}^H \mathbf{E}_{i,j} = \beta_{i,j} \mathbf{I}$ (including orthogonal designs) [1]

$$\mathbf{P}(\mathbf{S}_i \to \mathbf{S}_j) =$$

$$\frac{1}{2}\left\{1 - \sqrt{\frac{\gamma\beta_{i,j}}{2+\gamma\beta_{i,j}}} \sum_{l=0}^{MN-1} \binom{2l}{l} \left(\frac{1}{4+2\gamma\beta_{i,j}}\right)^l\right\}$$

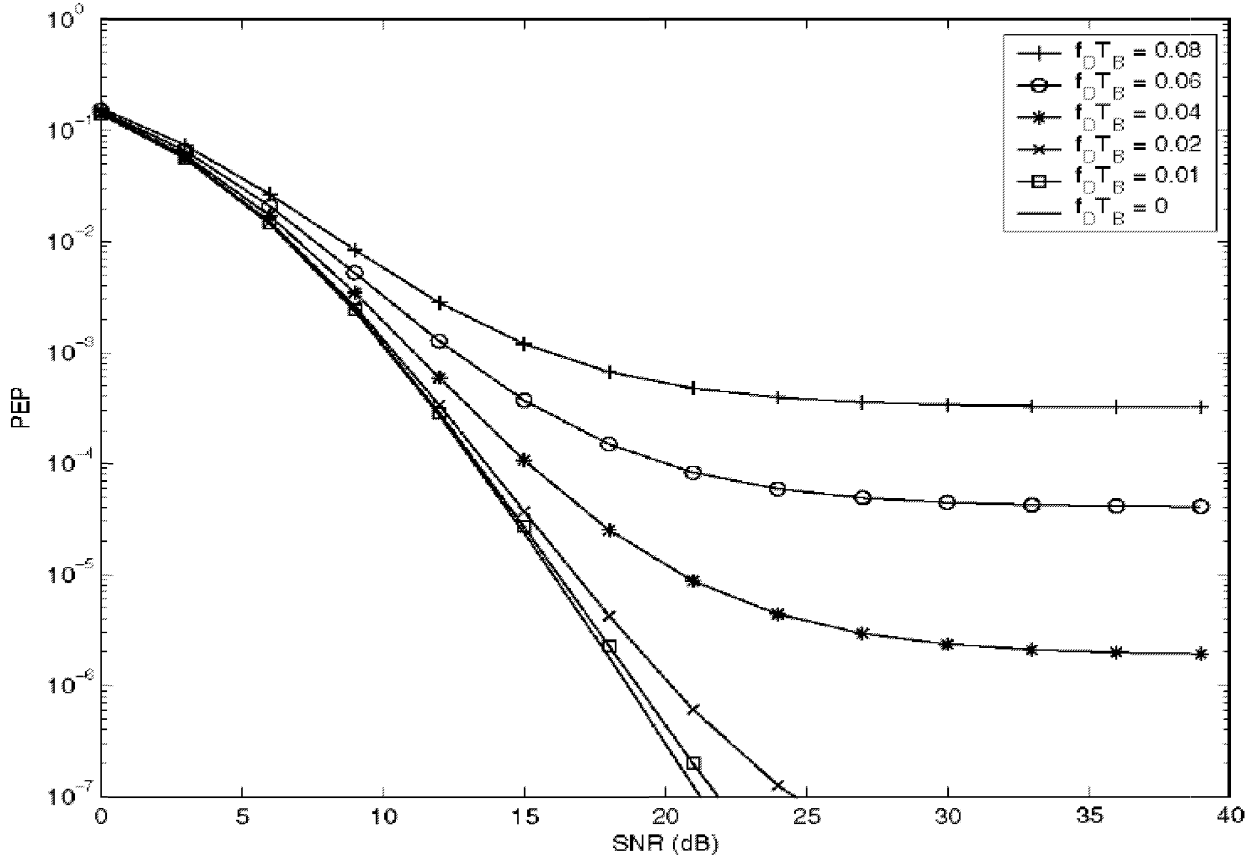
$$\gamma = \frac{\alpha^2 E_s^2}{2(E_s+\sigma_v^2)\left[E_s(1-\alpha^2)+\sigma_v^2\right] + 2\alpha^2 E_s \sigma_v^2}$$

Quasi static fading channel [1]



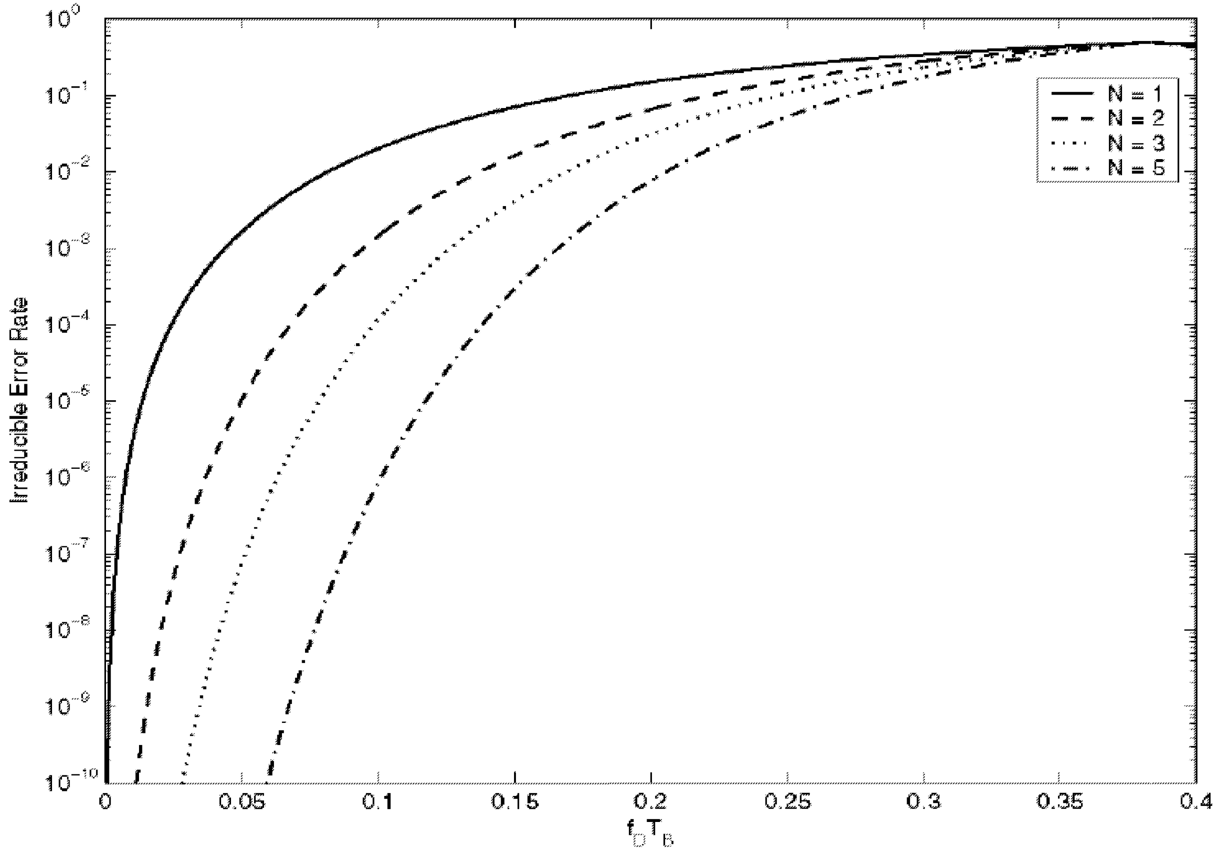
2×2 BPSK Alamouti design

Time varying fading channel with N=2 [1]



2×2 BPSK Alamouti design ($\alpha = J_0(2\pi f_D T_B)$)

Error floor in time varying fading channel [1]



2×2 BPSK Alamouti design

Conclusion

- The scheme is Differential STM, Code matrices are unitary, $\mathbf{h}_k \approx \mathbf{h}_{k-1}$, detection ML
- Exact PEP is analytically given
- To reduce PEP
  - Work on the code and generator constellation including non orthogonal design
  - Find another method to generate the transmission constellation
  - Work on the receiver side
    - Linear Prediction to estimate $\mathbf{h}_k$ from several previous $\mathbf{h}_l$ for $l = k-1, k-2, ..., k-p$
    - Multiple symbol detection
    - ...

Linear Prediction: Motivation

If the channel varies, there is an error floor for standard differential detection.
The Channel can be better estimated using more than one previous samples:

$$\mathbf{h}_k = \alpha_1 \mathbf{h}_{k-1} + \alpha_2 \mathbf{h}_{k-2} + \mathbf{n}_k$$

Where the variance of the noise is less than before and:

$$\mathbf{r}_k = \alpha_1 \mathbf{r}_{k-1} \mathbf{S}_k + \alpha_2 \mathbf{r}_{k-2} \mathbf{S}_k \mathbf{S}_{k-1} + noise$$

In this equation, $\mathbf{S}_{k-1}$ can be replaced by our estimation in the last period: $\hat{\mathbf{S}}_{k-1}$
$\Rightarrow$ Error propagation effect

Linear Prediction of order V (1)

If we consider an estimator of order V we have:

$$\mathbf{h}_k = \sum_{v=1}^{V} \alpha_v \mathbf{h}_{k-v} + noise$$

$$\mathbf{r}_k = \mathbf{S}_k \sum_{v=1}^{V} \left(\alpha_v \mathbf{r}_{k-v} \prod_{\mu=1}^{v-1} \mathbf{S}_{k-\mu} \right) + noise$$

ML detection gives:

$$\hat{\mathbf{S}}_k = \arg \min_{\mathbf{S}_k \in \mathcal{V}} \left\| \mathbf{r}_k - \mathbf{S}_k \sum_{v=1}^{V} \left(\alpha_v \mathbf{r}_{k-v} \prod_{\mu=1}^{v-1} \mathbf{S}_{k-\mu} \right) \right\|^2$$

Linear Prediction of order V (2)

Linear Prediction Decision Feedback Detection (LP-DFD) law:

$$\hat{\mathbf{S}}_k = \arg\max_{\mathbf{S}_k \in \mathcal{V}} \mathfrak{Re}\left[tr\left(\mathbf{S}_k \mathbf{r}_k^H \sum_{v=1}^{V} \alpha_v \mathbf{r}_{k-v} \prod_{\mu=1}^{v-1} \hat{\mathbf{S}}_{k-\mu}\right)\right]$$

Why does it work? Because we could write

$$\mathbf{r}_k = \alpha_1 \mathbf{S}_k \mathbf{X}_{k-1} \mathbf{h}_{k-1} + \alpha_2 \mathbf{S}_k \mathbf{X}_{k-1} \mathbf{h}_{k-2} + \cdots + \alpha_v \mathbf{S}_k \mathbf{X}_{k-1} \mathbf{h}_{k-v} + noise$$

$$\mathbf{r}_k = \alpha_1 \mathbf{S}_k \mathbf{X}_{k-1} \mathbf{h}_{k-1} + \alpha_2 \mathbf{S}_k \mathbf{S}_{k-1} \mathbf{X}_{k-2} \mathbf{h}_{k-2} + \cdots$$
$$+ \alpha_v \mathbf{S}_k \mathbf{S}_{k-1} \cdots \mathbf{S}_{k-v+1} \mathbf{X}_{k-v} \mathbf{h}_{k-v} + noise$$

$$\mathbf{r}_k = \alpha_1 \mathbf{S}_k \mathbf{r}_{k-1} + \alpha_2 \mathbf{S}_k \mathbf{S}_{k-1} \mathbf{r}_{k-2} + \cdots + \alpha_v \mathbf{S}_k \mathbf{S}_{k-1} \cdots \mathbf{S}_{k-v+1} \mathbf{r}_{k-v} + noise$$

Linear Prediction of order V (3)

So the **key relation** that makes LP DFD work is that we can write:

$$\mathbf{X}_{k-1}\mathbf{h}_{k-v} = \mathbf{S}_{k-1}\mathbf{S}_{k-2}\ldots\mathbf{S}_{k-v+1}\mathbf{X}_{k-v}\mathbf{h}_{k-v}$$

- The generator constellation is not necessarily diagonal.
- Generalization to fast fading channel where fading coefficients change for each channel use.

$$\mathcal{H}_k = \begin{bmatrix} \mathbf{h}_0[k] & \mathbf{h}_1[k] & \cdots & \mathbf{h}_{M-1}[k] \end{bmatrix}_{N\times M^2}$$

$$\mathcal{X}_k = \text{diag}\begin{bmatrix} \mathbf{X}_0[k] & \mathbf{X}_1[k] & \cdots & \mathbf{X}_{M-1}[k] \end{bmatrix}_{M^2\times M}$$

where $\mathbf{X}_i[k]$ is the i^{th} column of $\mathbf{X_k}$

Generalization to fast fading

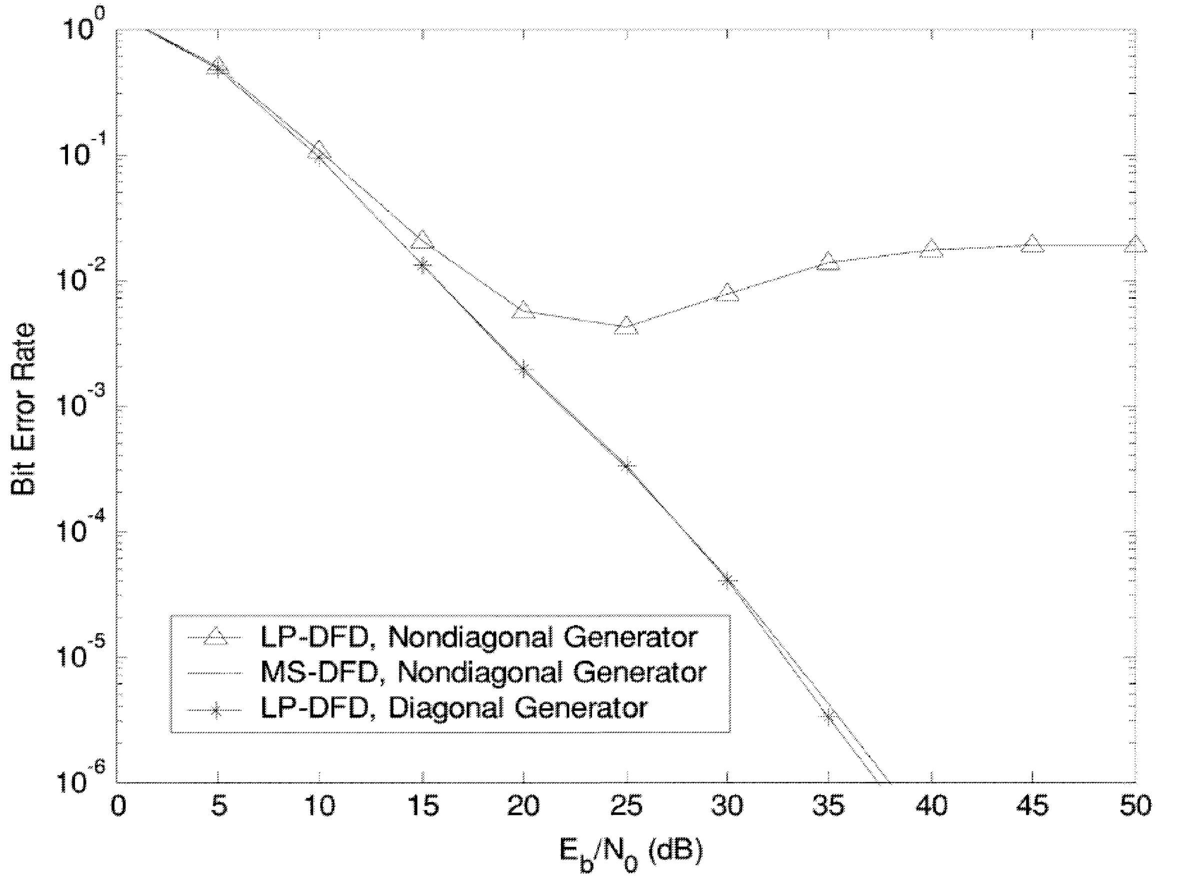
We can write now:

$$\mathbf{r}_{N\times M} = \mathcal{H}_{N\times M^2}\mathcal{X}_{M^2\times M} + \mathbf{v}_{N\times M}$$

Note that $\mathbf{X}_k = \mathbf{S}_{l(k)}\mathbf{X}_{k-1}$ does not necessarily guarantee $\mathcal{X}_k = \mathbf{S}_{l(k)}\mathcal{X}_{k-1}$ where channel is varying continuously

So the previous recursive key equation cannot be used.
Proposition: The above equation is valid if and only if the generator constellation is diagonal [3].
$\Rightarrow$ diagonal constellation can be used also for continuously varying channel.

LP performance for non diagonal constellation [3]



BPSK 3 Tx antennas design with $f_D T_B = 0.03, V = 3, N = 1$

Conclusion

- LP-DFD does not work for non diagonal generator constellation.
- Note that we can use a unitary initial matrix to distribute power evenly to Tx antennas and still use diagonal constellation.
- Another solution is to use Multiple Symbol Differential Detector based on Decision Feed-back Detector MS-DFD

Multiple Symbol Differential Detection - Motivation

- Each sent (received) symbol depends also on previous symbols.
- In order to take into account this correlation, sequence estimation can be justified.
- We look at $V + 1$ DSTM symbols in the receiver and we try to find with ML algorithm the most likely sequence of V matrices from the generator constellation.

Multiple Symbol Differential Detection - Basis [4]

Channel is varying continuously.

With proper definition of matrices, we can write the following equation for a block of $V+1$ DST symbols:

$$\bar{\mathbf{Y}}[k] = \bar{\mathcal{H}}[k]\bar{\mathcal{S}}[k] + \bar{\mathcal{N}}[k]$$

where
$\bar{\mathcal{H}}[k]$ contains $V+1$ times $M \times NM$ MIMO channel taps,
$\bar{\mathcal{S}}[k]$ contains $V+1$ transmission matrices each of size $M \times M$,
$\bar{\mathbf{Y}}[k]$ contains the received signals by N receive antennas during $V+1$ DST duration.

Decision rule

ML decoding maximizes the following probability function by testing the different matrices belonging to the transmit constellation.

$$f(\bar{\mathbf{Y}}[k]|\bar{S}[k]) = \frac{1}{\left(\pi^{M(V+1)} det(\mathbf{R}_{\bar{S}}^{-1}[k])\right)^N} exp\left\{-tr(\bar{\mathbf{Y}}[k]\mathbf{R}_{\bar{S}}^{-1}\bar{\mathbf{Y}}^H[k])\right\}$$

where $\mathbf{R}_{\bar{S}}$ is the autocorrelation matrix for $\bar{\mathbf{Y}}[k]$.

- Papers are published to simplify this decision rule
- In the case of diagonal generator constellation, significant simplifications are possible [4]

Further simplifications

- One way to simplify is to consider a sliding window. Each time just the last symbol is to be detected, the previous ones are the previous decision.
- For diagonal generator constellations, this is equal to LP-DFD [3]

Conclusion

- Conventional differential detector has 3 dB of degradation with respect to coherent detection
- Much more degradation is expected in continuous (fast) fading including error floor
- All presented methods try to approach the coherent receiver in a quasi static environment
- At the best, coherent detection performance in quasi static channel is hoped to be obtained.
- Time diversity is not exploited.
- The question is: how can we exploit the time diversity in fast fading.

How the *generator constellation* is constructed (1)

- Generator constellation is $\mathcal{V}$ where $\mathbf{S}_k \in \mathcal{V}$
- There are $L = 2^{N_T R}$ matrices in this set where R denotes the rate.
- These possibilities permit us to relate $N_T R$ bits of information to each matrix. So the rate is $R \;\; bpcu$.
- In diagonal constellation case all these matrices are diagonal and easily constructed:

$$\mathcal{V} = \{\mathbf{S}_0, \mathbf{S}_1, ..., \mathbf{S}_{L-1}\}$$

$$\mathbf{S}_l = \text{diag}\{e^{j2\pi u_0 l/L}, e^{j2\pi u_1 l/L}, ..., e^{j2\pi u_{N_T-1} l/L}\}$$

- The values of u_l (integers between 0 and L) should be optimized (see [5] for a table of optimized values)
- Note that the *transmission constellation* is not necessarily diagonal.

Mapping: relating information bits to matrix constellation

The goal is to reduce BER, so for each matrix we should look for the "nearest" matrices and then using *gray like* mapping.

References

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