Amplify-and-Forward Space-Time Coded Cooperation via Incremental Relaying

Behrouz Maham and Are Hjørungnes

UniK – University Graduate Center, University of Oslo
Instituttveien-25, N-2027, Kjeller, Norway
behrouz@unik.no, arehj@unik.no

***Abstract*— User cooperation can provide spatial transmit diversity gains, enhance coverage, and potentially increase capacity. In this paper, *incremental relaying* protocol is developed. This protocol uses a limited feedback from the destination to transmitters to choose between direct transmission or cooperation. The incremental relaying protocol is examined in a space-time coded cooperation under the amplify-and-forward model. The exact bit error rate of the system is derived. Furthermore, the bit error rate of amplify-and-forward based space-time cooperation is derived, when the source-destination link is considered in both phases. Simulations demonstrate that the proposed scheme can substantially improve the spectral efficiency of the system comparing to the conventional two-phased cooperative schemes. Moreover, it is shown that the proposed scheme can achieve an outstanding coding gain in comparison to direct transmission.**

I. Introduction

Space-time coding (STC) has received a huge attention in the last years as a way to increase capacity and/or reduce the transmitted power necessary to achieve a target bit error rate (BER) using multiple antenna transceivers. More recently, cooperative diversity techniques have been introduced to improve the spectral and power efficiency of the wireless networks [1]-[3]. Cooperative diversity allows a collection of radios to relay signals for each other and effectively create a virtual antenna array for combating multipath fading in wireless channels. The attractive feature of these techniques is that each node is equipped with only one antenna, creating a virtual antenna array. This property makes them outstanding for deployment in cellular mobile devices as well as in ad-hoc mobile networks, which have problem with exploiting multiple-antenna due to the size limitation of the mobile terminals.

On the other hand, conventional relaying protocols, i.e., fixed and dynamic relaying (see, e.g., [2]) can make inefficient use of the degrees of freedom of the channel, especially for high rates, since the relays repeat all the time. Authors in [3] proposed *incremental relaying* protocols for repetition-based cooperation that exploit limited feedback from the destination terminal, e.g., a single bit indicating the success or failure of the direct transmission. This method can dramatically improve spectral efficiency over fixed and selection relaying. These incremental relaying protocols can be viewed as extensions of incremental redundancy, or hybrid automatic-repeat-request (ARQ), to the relay context. In [4], cyclic redundancy check (CRC) bits are employed to enable detection of decoding errors at the destination. If the destination node determines from the CRC bits that the direct copy during Phase I has been decoded correctly, it will send back an ACK for the users to start transmitting new information; otherwise, a NACK will inform them to send the relay copy during Phase II.

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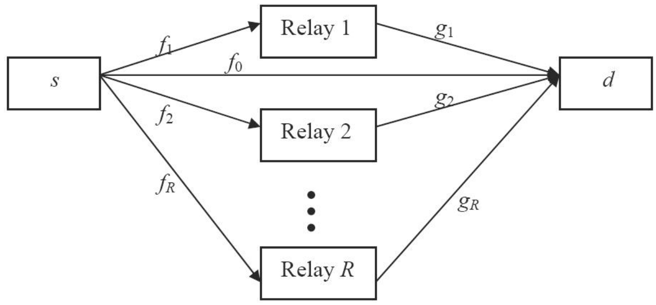

Fig. 1. Wireless relay network consisting of a source s, a destination d, and R relays.

The main contributions of this paper are as follows. We develop an incremental relaying protocol based on distributed space-time processing in amplify-and-forward (AF) mode. We utilize the instantaneous signal to noise ratio (SNR) to enable detection of decoding errors at the destination. If the source-destination SNR is not sufficiently high for errorless direct transmission, the feedback requests that the relays amplify-and-forward what they received from the source with space-time coded cooperation. Moreover, the bit error rate of amplify-and-forward based space-time cooperation and its incremental relaying version are derived, when the source-destination link is considered in both phases.

This paper is organized as follows: In Section II, the system model is given. The performance analysis of an incremental relaying AF-based distributed space-time system is presented in Section III. The BER expression for AF-based distributed space-time system in which the source-destination link contributes in both phases is also derived in Section III. In Section IV, simulation results are given. Finally, conclusions are presented in Section V.

II. System Model

Consider a network in Fig. 1 consisting of a source denoted s, one or more relays denoted $r = 1, 2, \ldots, R$, and one destination denoted d. It is assumed that each node is equipped with a single antenna. We consider symmetric channels and denote the source-to-destination, source-to-*r*th relay, and *r*th relay-to-destination links by f_0, f_r, and g_r, respectively. Suppose each link has Rayleigh fading, independent of the others. Therefore, f_0, f_r, and g_r are i.i.d. complex Gaussian random variables with zero-mean and variances σ_0^2, σ_f^2, and σ_g^2, respectively. The source node, s, transmits a signal $\boldsymbol{s} = [s_1, \ldots, s_T]^t$, consisting of *T* symbols to the destination and *all* relays. In the cooperation mode, i.e., when source-destination link is in bad erroneous condition, incremental relaying scheme requires the second phase of transmission using relays; otherwise, the source node sends new *T* symbols. We assume the following normalization: $E\{\boldsymbol{s}^H \boldsymbol{s}\} = 1$. Assuming that f_0 is not varying during *T* successive intervals, the received $T \times 1$ signal at the destination from the source can be written as

$$\boldsymbol{x}_0 = \sqrt{P_1 T} f_0 \boldsymbol{s} + \boldsymbol{w}_0, \tag{1}$$

where $P_1 T$ is the average total transmitted energy in *T* intervals, and $\boldsymbol{w}_0$ is a $T \times 1$ complex zero-mean white Gaussian noise vector with component-wise variance N_0.

For the case that the signal-to-noise (SNR) of the source-destination link is low, incremental relaying scheme requires the second phase of transmission using relays. Therefore, the relays are employed to increase the diversity order. The received $T \times 1$ signal at the *r*th relay can be written as

$$\boldsymbol{r}_r = \sqrt{P_1 T} f_r \boldsymbol{s} + \boldsymbol{v}_r, \tag{2}$$

where $\boldsymbol{v}_r$ is a $T \times 1$ complex zero-mean white Gaussian noise vector with variance of N_1. Under amplify-and-forward, each relay scales its received signal, i.e.,

$$\boldsymbol{x}_r = \rho_r \boldsymbol{r}_r, \tag{3}$$

where ρ_r is the scaling factor at relay *r*. When there is no instantaneous channel state information (ICSI) at the relays, but statistical channel state information (SCSI) is known, a useful constraint is to ensure that a given average transmitted power is maintained. That is,

$$\rho_r^2 = \frac{P_2}{\sigma_f^2 P_1 + N_1}, \tag{4}$$

where P_2 is the average transmitted power at relay *r*, such that all relays transmits with the same power.

DSTC, proposed in [8], uses the idea of linear dispersion space-time codes of multiple-antenna systems. In this system, the $T \times 1$ received signal at destination can be written as

$$\boldsymbol{x} = \sum_{r=1}^{R} g_r \boldsymbol{A}_r \boldsymbol{x}_r + \boldsymbol{w}, \tag{5}$$

where $\boldsymbol{x}_r$ is given in (3), $\boldsymbol{w}$ is a $T \times 1$ complex zero-mean white Gaussian noise vector with component-wise variance of N_2, and $\boldsymbol{A}_r$, $r = 1, \ldots, R$, are $T \times T$ unitary matrices. $\boldsymbol{A}_r \boldsymbol{s}$, $r = 1, \ldots, R$, must describe columns of a proper $T \times 1$ space-time code, such as the codes given in [7] and [5]. Furthermore, depending on contribution of the source node in two phases, we can categorized the relay-assisted network into four groups, which is shown in Table I. In Protocol I, the source contributes in two phases of transmission. Thus, destination utilized all degrees of freedoms in the network. Using Protocol II is useful in cases that the source node is involved in receiving information from another node in the network in Phase II. Therefore, it cannot be used to transmit data in the second phase. Similarly, in Protocol III, destination node may be engaged in receiving power from another node in the first phase. Thus, the transmitted signals in the first phase are just received by the relays and will be buffered for the next phase of transmission. In Protocol II, the source node remains silent in the second phase. This means that in the cases that the source is located far from the destination, this protocol outperforms in the sense of power efficiency comparing to Protocol I and III. Finally, Protocol IV is the simplest case in which the source-destination link is not considered in none of phases. This protocol is employed e.g., in [5] and [7].

TABLE I

PROTOCOLS IN COOPERATIVE SYSTEMS BASED ON THE AVAILABLE CHANNEL DEGREES OF FREEDOM

Time Slot Protocol	I	II	III	IV
1	$s \to r, d$	$s \to r, d$	$s \to r$	$s \to r$
2	$s, r \to d$	$r \to d$	$s, r \to d$	$r \to d$

III. PERFORMANCE ANALYSIS

In this section, we will derive the exact bit error rate (BER) of the AF space-time coded cooperation under the incremental relaying protocol when maximum-likelihood decoding is used.

In [3], for computing the outage probability, it is assumed that if the condition $|f_0|^2 > \gamma$ is satisfied, destination can decode the received signals errorless, where γ is defined as

$$\gamma = \frac{2^\eta - 1}{\text{SNR}_0}, \tag{6}$$

where η is the normalized transmission rate, or spectral efficiency. Using (1), we can write SNR_0 as $\text{SNR}_0 = P_1/N_0$. Thus, we use γ as a threshold to choose between direct transmission and cooperation. In outage probability analysis performed in [3], it is ideally assumed errorless detection at the destination when the condition $|f_0|^2 > \gamma$ is satisfied. In this work, we also consider the errors in direct transmission phase.

Using the general law of probability, the bit probability of error of the proposed incremental relaying protocol at the destination can be written as

$$\begin{aligned} P_{e,\text{IAF}} &= \Pr\left(|f_0|^2 \le \gamma\right) P_{e,\text{AF}} + \Pr\left(|f_0|^2 > \gamma\right) P_{e,\text{direct}} \\ &= \left(1 - e^{-\frac{\gamma}{\sigma_0^2}}\right) P_{e,\text{AF}} + e^{-\frac{\gamma}{\sigma_0^2}} P_{e,\text{direct}}, \end{aligned} \tag{7}$$

where $P_{e,\text{direct}}$ is the conditional probability of error of the source-destination link. $P_{e,\text{AF}}$ is the probability of error for the space-time coded cooperation in AF mode, which is derived in [6, Eq. (22)]. The conditional BER of the source-destination link is

$$P_{e,\text{direct}} = \text{E}\left\{ c\, Q\left(\sqrt{g\, \text{SNR}_0\, |f_0|^2}\right) \Big|\, |f_0|^2 > \gamma \right\}, \tag{8}$$

where the parameters c and g depend on the modulation type. Thus, using the method of Moment Generating Function (MGF), we can rewrite (8) as

$$\begin{aligned} P_{e,\text{direct}} &= \int_{\gamma}^{\infty} c\, Q\left(\sqrt{g\,\text{SNR}_0\,\alpha}\right) p_0(\alpha) d\alpha \\ &= \int_{\gamma}^{\infty} \frac{c}{\pi} \int_{0}^{\pi/2} \exp\left(-\frac{g\,\text{SNR}_0\,\alpha}{\sin^2\phi}\right) d\phi\, p_0(\alpha) d\alpha, \end{aligned} \tag{9}$$

where $\alpha = |f_0|^2$ has an exponential distribution. Hence, using the following integral,

$$\int_{\gamma}^{\infty} e^{s\,\alpha} p_0(\alpha) d\alpha = \int_{\gamma}^{\infty} e^{s\,\alpha} \frac{1}{\sigma_0^2} e^{-\frac{\alpha}{\sigma_0^2}} d\alpha = \frac{e^{-\frac{\gamma}{\sigma_0^2}\left(1-s\sigma_0^2\right)}}{1-s\sigma_0^2}, \tag{10}$$

we can express (9) as

$$P_{e,\text{direct}} = \frac{c}{\pi} \int_{0}^{\pi/2} \frac{\exp\left(-\frac{\gamma}{\sigma_0^2}\left(1+\frac{g\,\text{SNR}_0\,\sigma_0^2}{\sin^2\phi}\right)\right)}{\left(1+\frac{g\,\text{SNR}_0\,\sigma_0^2}{\sin^2\phi}\right)} d\phi. \tag{11}$$

The AF space-time cooperation error, $P_{e,\text{AF}}$, for any full-rate and full-diversity space-time codes is calculated analytically in [6]. Hence, for Protocol III and IV we have

$$P_{e,\text{AF}} = \frac{c}{\pi} \int_{0}^{\frac{\pi}{2}} M_0\left(-\frac{g\,\text{SNR}_0}{\sin^2\phi}\right) \left(M\left(-\frac{g\,\text{SNR}}{\sin^2\phi}\right)\right)^R d\phi, \tag{12}$$

and

$$P_{e,\text{AF}} = \frac{c}{\pi} \int_{0}^{\frac{\pi}{2}} \left(M\left(-\frac{g\,\text{SNR}}{\sin^2\phi}\right)\right)^R d\phi, \tag{13}$$

respectively, where parameters c and g are represented as

$$c_{\text{QAM}} = 4\frac{\sqrt{M}-1}{\sqrt{M}\log_2 M},\; c_{\text{PSK}} = \frac{2}{\log_2 M},$$
$$g_{\text{QAM}} = \frac{3}{M-1},\; g_{\text{PSK}} = 2\sin^2\left(\frac{\pi}{M}\right),$$

and SNR can be written as

$$\text{SNR} = \frac{\frac{P_1 P_2}{\sigma_f^2 P_1 + N_1}}{R\frac{P_2}{\sigma_f^2 P_1 + N_1}\sigma_g^2 N_1 + N_2}. \tag{14}$$

$M_0(\cdot)$ in (12) is the moment generating function of the direct source-destination link, which is the MGF of an exponential random variable. $M(\cdot)$ in (12) and (13) is the MGF of the equivalent channel passing from the *r*th relay and its closed-form expression is obtained in [6, Eq. (24)]. Note that these analytical results are achieved by assuming that source-to-relays channels and relays-to-destination channels have a same length.

Assuming the usage of full-diversity distributed space-time codes in AF mode (see, e.g., [5] and [7]), $P_{e,\text{IAF}}$ is computed by substituting $P_{e,\text{direct}}$ from (11) and $P_{e,\text{AF}}$ from (12) and (13) for Protocols IV and III, respectively.

For Protocols I and II, we are going to obtain the expressions for BER. We first consider Protocol I, which uses all degrees of freedom of the channels. Other mentioned protocols are the special cases of Protocol I. Protocols I and II utilized the time diversity achieved by considering the first phase transmission by decoder at the destination. In this way, the full time diversity is achieved as well as spatial diversity. In the case that the coherence time of the channel is assumed to be T, Protocol I attains the full spatial-time diversity of order $R+2$. Using orthogonal space-time codes, Maximum Likelihood (ML) decoding will be equivalent to Maximum Ratio Combining (MRC) method. Moreover, for utilizing the transmitted information by the source in the first phase in the decoding process of destination, MRC method can be used to obtain the time diversity.

Here, we are going to derive the BER formula of full-diversity and full-rate space-time codes in AF mode under Protocol I. The method we are using here is the same as [6], which is used moment generating function method.
The conditional BER of Protocol I, with *R* relays, can be written as

$$P_b\left(R|\{f_r\}_{r=0}^R, \{g_r\}_{r=1}^R\right) = c\,Q\left(\sqrt{g\,\text{SNR}^{\text{out}}}\right), \tag{15}$$

where SNR^{out} can be written as

$$\text{SNR}^{\text{out}} = \text{SNR}_{1,0}|f_0^{(1)}|^2 + \text{SNR}_{2,0}|f_0^{(2)}|^2 + \text{SNR}\sum_{r=1}^{R}|f_r\,g_r|^2, \tag{16}$$

where $f_0^{(1)}$ and $f_0^{(2)}$ are the channel coefficients of the source-destination direct path during Phase I and Phase II, respectively. We can write $\text{SNR}_{1,0}$ as

$$\text{SNR}_{1,0} = \frac{P_1}{N_0}, \tag{17}$$

and the coefficient $\text{SNR}_{2,0}$ can be written as

$$\text{SNR}_{2,0} = \frac{P_2}{R\frac{P_2}{\sigma_f^2 P_1 + N_1}\sigma_g^2 N_1 + N_2}. \tag{18}$$

If we define $\gamma_{1,0}$, $\gamma_{2,0}$, and γ_r are as

$$\gamma_{1,0} = |f_0^{(1)}|^2,\; \gamma_{2,0} = |f_0^{(2)}|^2,\; \gamma_r = |f_r\,g_r|^2,$$

we can represent SNR^{out} as

$$\text{SNR}^{\text{out}} = \text{SNR}_{1,0}\gamma_{1,0} + \text{SNR}_{2,0}\gamma_{2,0} + \text{SNR}\sum_{r=1}^{R}\gamma_r, \tag{19}$$

where $\gamma_{1,0}$ and $\gamma_{2,0}$ have an exponential distribution, and γ_r has an distribution as [6, Eq. (18)].

Since the γ_rs are independent, using the moment generating function approach, assuming that we can achieve full diversity, we can get

$$\begin{aligned} P_e(R) &= \int_{0;\,(R+2)-\text{fold}}^{\infty} P_b\left(R|\{f_r\}_{r=0}^R, \{g_r\}_{r=1}^R\right) p_0(\gamma_{1,0}) d\gamma_{1,0} \\ &\quad \cdot p_0(\gamma_{2,0}) d\gamma_{2,0} \prod_{r=1}^{R} \left(p(\gamma_r)\, d\gamma_r\right) \\ &= \frac{c}{\pi} \int_{0}^{\frac{\pi}{2}} M_{1,0}\left(-\frac{g\,\text{SNR}_{1,0}}{\sin^2\phi}\right) M_{2,0}\left(-\frac{g\,\text{SNR}_{2,0}}{\sin^2\phi}\right) \\ &\quad \cdot \left[M\left(-\frac{g\,\text{SNR}}{\sin^2\phi}\right)\right]^R d\phi, \end{aligned} \tag{20}$$

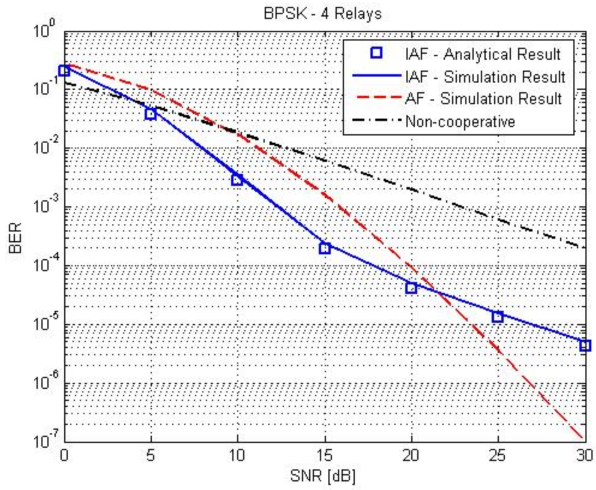

Fig. 2. The average BER curves versus SNR of a distributed space-time system with the employment of different protocols.

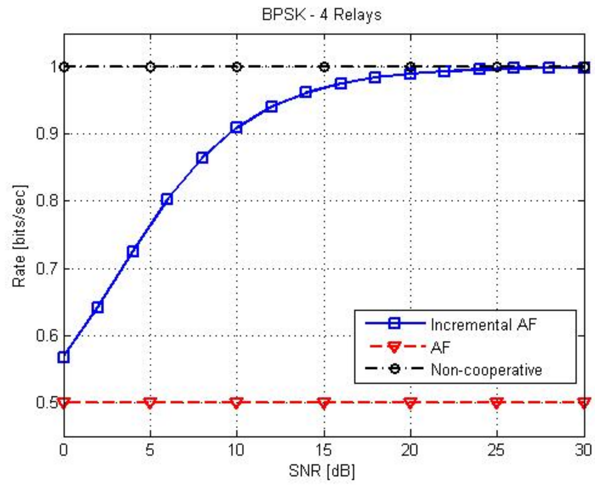

Fig. 3. The average normalized rate of a distributed space-time coded cooperation system with the employment of different protocols.

where $M_{1,0}(\cdot)$, $M_{2,0}(\cdot)$, and $M(\cdot)$ are the moment generating functions of random variables $\gamma_{1,0}$, $\gamma_{2,0}$, and γ_r, respectively. Note that, using (20), the BER formulas for Protocols II, III, and IV can be easily obtained. For example, for Protocol II in which the source node does not contribute in the second phase, we can put $\mathrm{SNR}_{2,0}$ as zero in linear scale (not dB). Thus, $M_{2,0}(\cdot)$ function would be equal to 1 and the BER expression for Protocol II is obtained.

IV. Simulation Results

In this section, the performances of incremental relaying using AF distributed space-time codes are studied through simulations. The number of relays supposed to be $R = 4$. We utilized distributed version of GABBA codes, introduced in [5], as practical full-diversity distributed space-time codes. We also assumed BPSK modulation.

In Fig. 2, the BER performance of the incremental relaying protocol is compared with the fixed AF protocol and direct transmission. We use AF space-time cooperation based on [8]. One can observe from Fig. 2 that the incremental AF (IAF) protocol outperforms the fixed AF protocol with full diversity space-time codes for low SNR conditions. For example, comparing two cooperative curves in Fig. 2 demonstrates over 3 dB gain in SNR at the BER of 10^{-3}, using IAF protocol. Since the IAF curve is in parallel with the non-cooperative curve in high SNR conditions, the IAF protocol could not reach full diversity as the fixed AF protocol. However, IAF can achieve an outstanding coding gain comparing the direct non-cooperative transmission. Furthermore, Fig. 2 confirms that the analytical results attained in Section II for finding BER have the same performance as simulations. One can observe that at low SNR scenarios, non-cooperative system outperform from the AF space-time coded cooperation. This is because the fact that in Protocol IV the direct transmission is not considered.

Fig. 3 illustrates the spectral efficiency curves as functions of the average SNR per symbol for different protocols. It is obvious that spectral efficiency of AF space-time codes is half of the direct transmission duo to its two-phased transmission nature. Comparing two cooperative protocol curves, one observes that incremental AF protocol can considerably improve the spectral efficiency of the system. It can be seen that in high SNR scenarios the incremental AF achieves the maximum value of the spectral efficiency in the expense of loosing the full-diversity benefit of fixed AF protocol.

In Fig. 4, protocols in Table I are employed in the Incremental Relaying AF-based space-time cooperation introduced in Section II. It can be seen that Protocol I outperforms from the other protocols in low SNR regime. However, all of the employing protocols have the same performance in high SNR due to prevalence of the direct source-destination link in high SNRs.

Figures 5 and 6, demonstrate the diversity-multiplexing tradeoff in Incremental Relaying AF-based space-time cooperative system. By increasing the SNR threshold for choosing between non-cooperative transmission and cooperative transmission, the higher diversity gain will be achieved, while the spectral efficiency of the system is reduced.

Finally, Fig. 7 compares the performance of protocols in Table I. The full-diversity, full-rate distributed space-time codes [5] used under AF model. Since we have assumed the same distance for source-to-destination, source-to-relay, and relay-to-destination links, the performance of Protocol II is better than Protocol III. Moreover, as we expect, Protocol I has the best performance comparing to the other protocols.

V. Conclusion

In this paper, we proposed using incremental relaying protocol in AF based space-time cooperation. In our scheme, instantaneous signal to noise ratio (SNR) utilized to decision between direct transmission or two-phase cooperative transmission using available relays. Our results showed that using this scheme, spectral efficiency of the proposed system highly

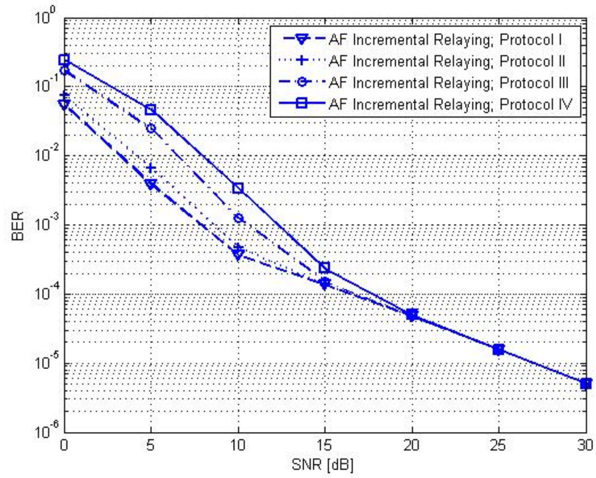

Fig. 4. The average BER curves versus SNR of the Incremental Relaying AF-based space-time cooperation when the employment of different protocols in TABLE I.

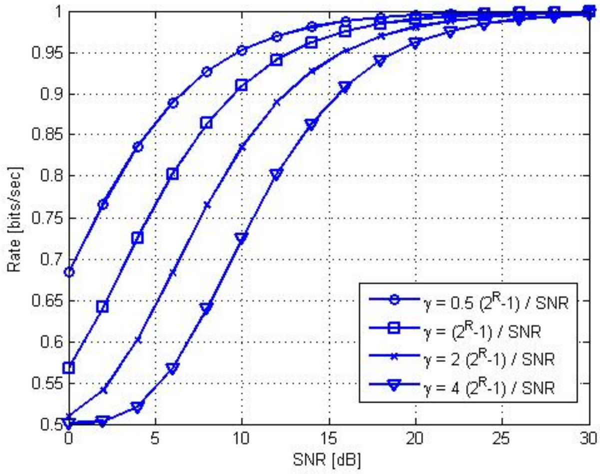

Fig. 6. The average normalized rate of the Incremental Relaying AF-based space-time cooperation when different threshold on SNR employed.

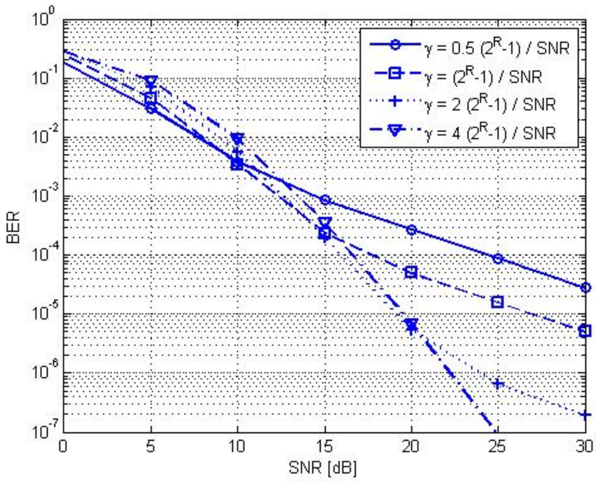

Fig. 5. The average BER curves versus SNR of the Incremental Relaying AF-based space-time cooperation when different threshold on SNR employed.

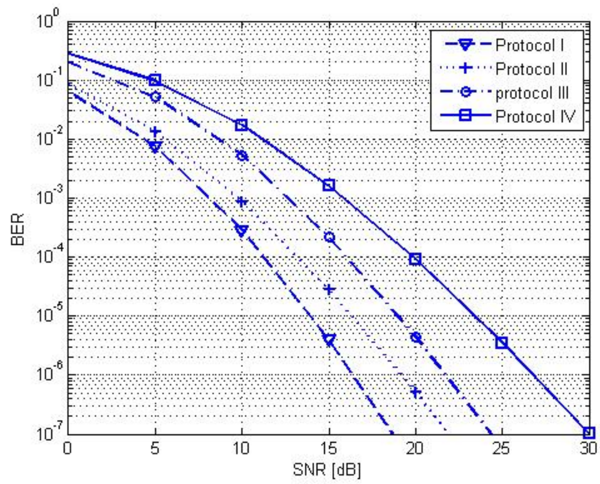

Fig. 7. The average BER curves versus SNR of AF-based space-time cooperation with the employment of different protocols in TABLE I.

increases comparing to AF space-time cooperation. Moreover, simulations demonstrated that a considerable coding gain can be achieved in respect to the non-cooperative direct transmission scenario. Furthermore, the BER expression for the AF space-time cooperation based on incremental relaying was derived. The BER expression for AF-based distributed space-time system in which source-destination link contributes in both phases was also derived.

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