# Cooperative techniques

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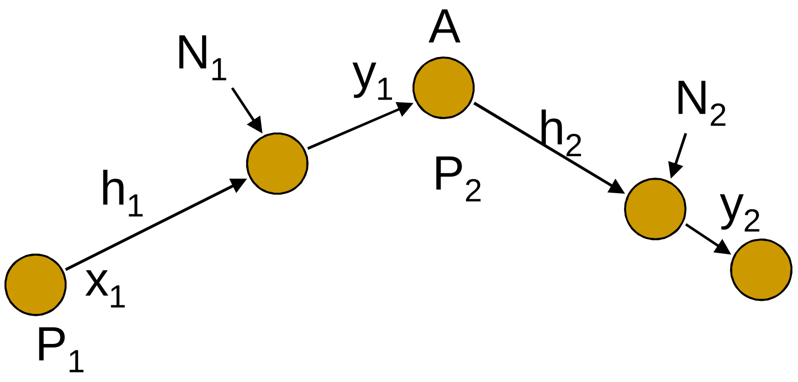
# Plan

- Amplify and Forward (AF) Scheme
    - Two-hop serial and parallel
    - Multi-hop serial and parallel
    - SNR calculations with MRC at the destination
    - SER calculation for high SNR
    - All Participate AF (AP-AF) versus Selection AF (S-AF)
- Cooperative multiple access transmission using precoding vector

# Simple A&F Scheme



-The gain at the relay is A

-The noises are CN(0,$N_i$)

-Suppose the power of the constellation is normalized to unity

1- Hasna, M.O.; Alouini, M.-S., "End-to-end performance of transmission systems with relays over Rayleigh-fading channels," Wireless Communications, IEEE Transactions on , vol.2, no.6, pp. 1126-1131, Nov. 2003

2- A.Ribeiro, X. Cai, G. Giannakis, “Symbol Error probabilities for General Cooperative Links”, IEEE trans on commun., May 2005.

# System Model

$$y_1 = \sqrt{P_1} h_1 x_1 + n_1$$

$$y_2 = A h_2 \left( \sqrt{P_1} h_1 x_1 + n_1 \right) + n_2$$

The transmitted power at the relay is $P_2 = A^2 \left( P_1 |h_1|^2 + N_1 \right)$

The SNR at each stage is defined as follows: $\gamma_1 \triangleq \frac{P_1 |h_1|^2}{N_1}$ $\gamma_2 \triangleq \frac{P_2 |h_2|^2}{N_2}$

The SNR at the output will be:

$$\boxed{\gamma = \frac{A^2 |h_2|^2 \, p_1 |h_1|^2}{A^2 |h_2|^2 \, N_1 + N_2} = \frac{\gamma_1 \gamma_2}{1 + \gamma_1 + \gamma_2}}$$

And this is independent of A.

# SNR at the Output

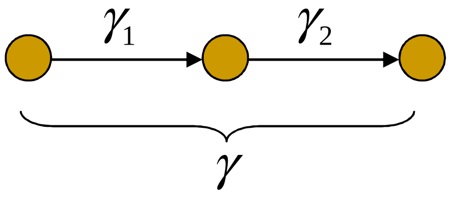
Approximation (high SNR): $\gamma = \dfrac{\gamma_1\gamma_2}{1+\gamma_1+\gamma_2} \approx \dfrac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}$ $\qquad \dfrac{1}{\gamma} \approx \dfrac{1}{\gamma_1} + \dfrac{1}{\gamma_2}$

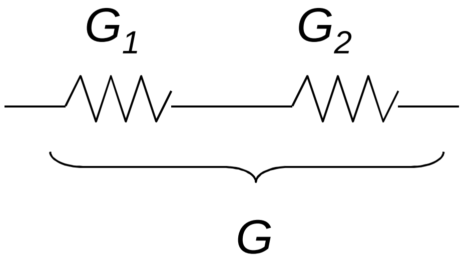
Another way of approximation: $P_2 = A^2\left(P_1\left|h_1\right|^2 + N_1\right) \approx A^2 P_1\left|h_1\right|^2$

If we replace this quantity in SNR calculation we will obtain the same equation as above:

$$\frac{1}{\gamma} \approx \frac{1}{\gamma_1} + \frac{1}{\gamma_2}$$

Therefore, a serial link can be imagined as the cascade of two resistances and the SNRs can be considered as the conductance of each link. The total conductance (SNR) can be found using the above formula.





# Optimum Gain A

The question is “what is the optimum gain to be applied in the relay ?” Supposing the approximation stated before, the SNR should be maximized, or the following function should be minimized by optimally distribution of total power:

$$\frac{1}{\gamma} \approx \frac{1}{\gamma_1} + \frac{1}{\gamma_2}$$

The constraint is : $P_1 + P_2 = P = cte$

Considering that the power is distributed between the relay and the source:

$$P_1 = \beta P, \qquad P_2 = (1-\beta)P$$

We obtain: $$\frac{1}{\gamma} = \frac{N_1}{P_1 |h_1|^2} + \frac{N_2}{P_2 |h_2|^2} = \frac{N_1}{\beta P |h_1|^2} + \frac{N_2}{(1-\beta) P |h_2|^2}$$

Derivation with respect to beta equal to zero:

# Power Allocation

$$\frac{\partial}{\partial\beta}\left(\frac{1}{\gamma}\right)=\frac{N_1}{P|h_1|^2}\frac{-1}{\beta^2}+\frac{N_2}{P|h_2|^2}\frac{1}{(1-\beta)^2}=0$$

Simplifying the above equation gives:

$$\beta=\frac{\sqrt{\frac{|h_2|^2}{N_2}}}{\sqrt{\frac{|h_1|^2}{N_1}}+\sqrt{\frac{|h_2|^2}{N_2}}}$$

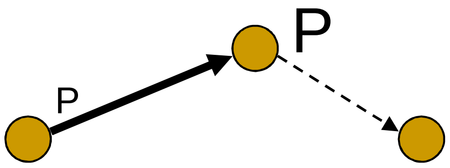
In the limit cases, if h$_2$ goes to zero, beta is small, which means that the power should be used in the second link. In other words, it is not useful to expend power in the first link knowing that the second link cannot transmit the information.

Defining the normalized SNR as: $\gamma'_1 \triangleq \frac{|h_1|^2}{N_1}$ and $\gamma'_2 \triangleq \frac{|h_2|^2}{N_2}$

$$\beta=\frac{\sqrt{\gamma'_2}}{\sqrt{\gamma'_1}+\sqrt{\gamma'_2}} \quad\Rightarrow\quad P_1=\frac{\sqrt{\gamma'_2}}{\sqrt{\gamma'_1}+\sqrt{\gamma'_2}}P \text{ and } P_2=\frac{\sqrt{\gamma'_1}}{\sqrt{\gamma'_1}+\sqrt{\gamma'_2}}P \Rightarrow \frac{P_1}{P_2}=\sqrt{\frac{\gamma'_2}{\gamma'_1}}$$

$$\boxed{A^2\approx\frac{1}{|h_1||h_2|}\frac{\sqrt{N_1}}{\sqrt{N_2}}}$$

(this is obtained Supposing $P_2\approx A^2P_1|h_1|^2$)



# Power allocation

- If at the relay the information of the forwarding channel is not available ($h_2$ is unknown), then $h_2$ can be replaced by its mean and it is assumed that $N_1$=$N_2$.

$$A^2 \approx \frac{1}{|h_1|.mean\{|h_2|\}} \qquad h_2 = h_{2r} + jh_{2i} \sim CN(0,1)$$

$|h_2|$ is a Rayleigh distributed RV $p_{h_2}(h) = \frac{h}{\sigma^2} e^{-h^2/2\sigma^2}$ with $\sigma^2 = 0.5$

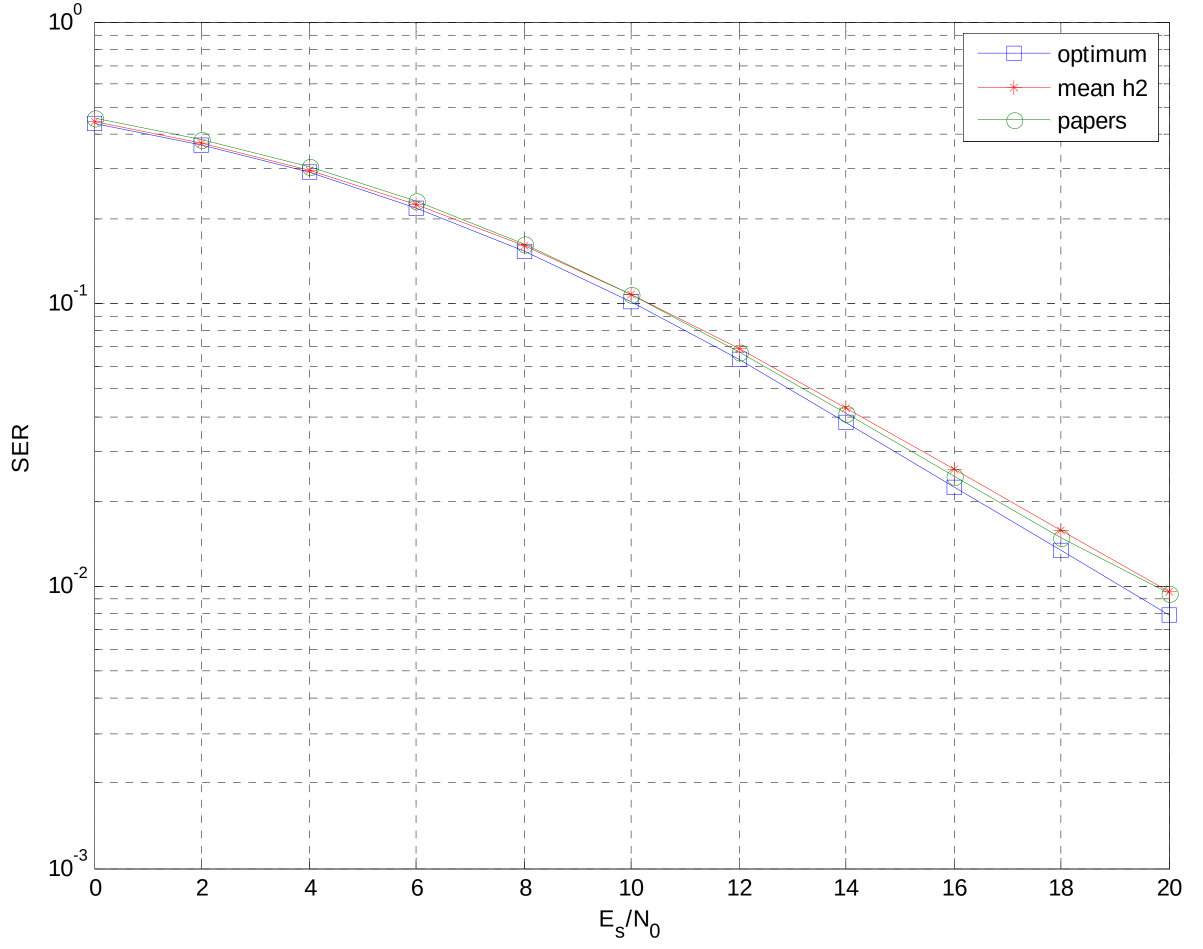
The mean of $|h_2|$ is $\sqrt{2\sigma^2}\Gamma(3/2) = \frac{1}{2}\sqrt{\pi}$

$$\boxed{A^2 \approx \frac{2}{\sqrt{\pi}|h_1|}}$$

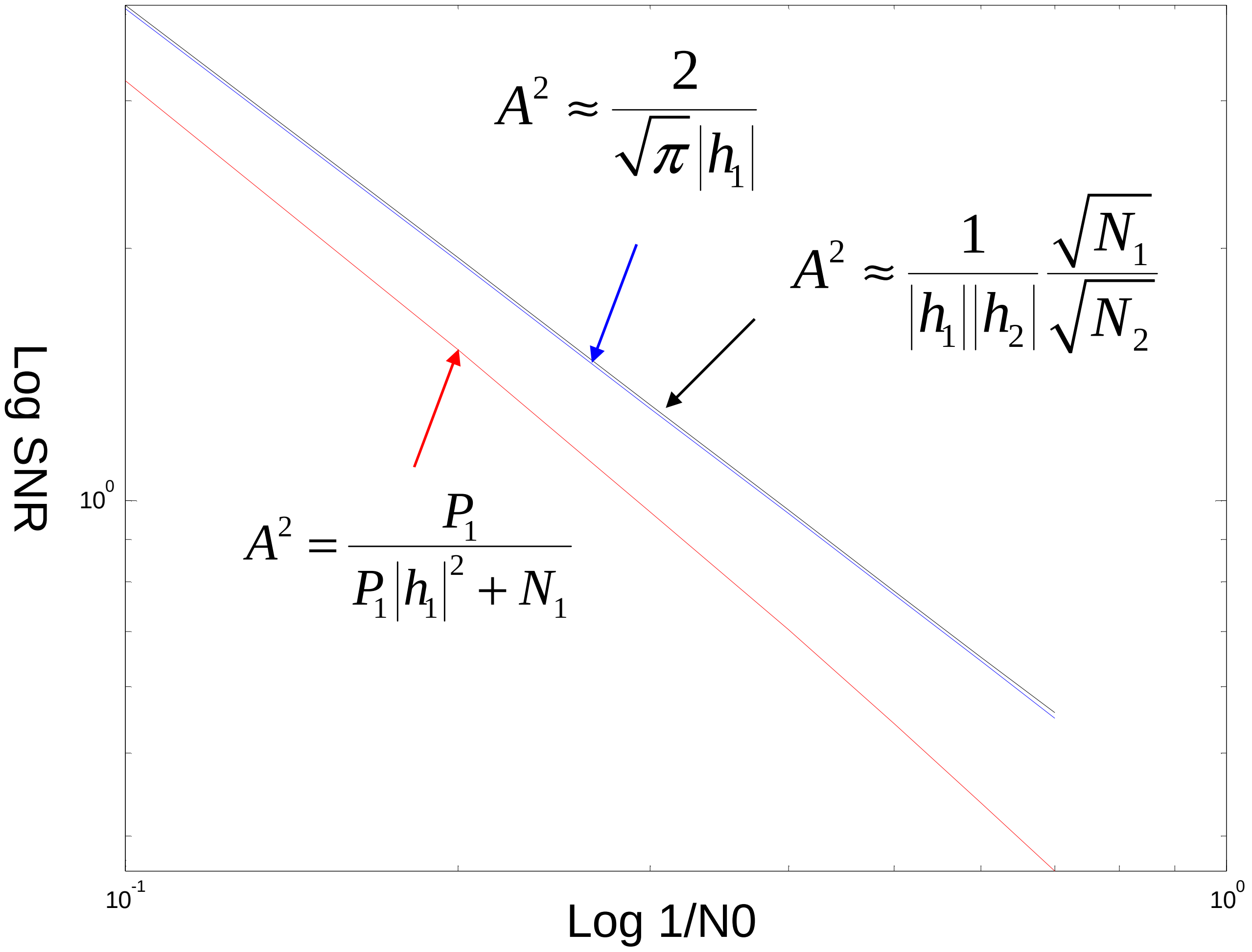
In the papers, a convenient solution is to choose the gain in order that the relay power to be equal to the original transmitted power. This is not the best solution !

$$A^2 = \frac{P_1}{P_1|h_1|^2 + N_1}$$

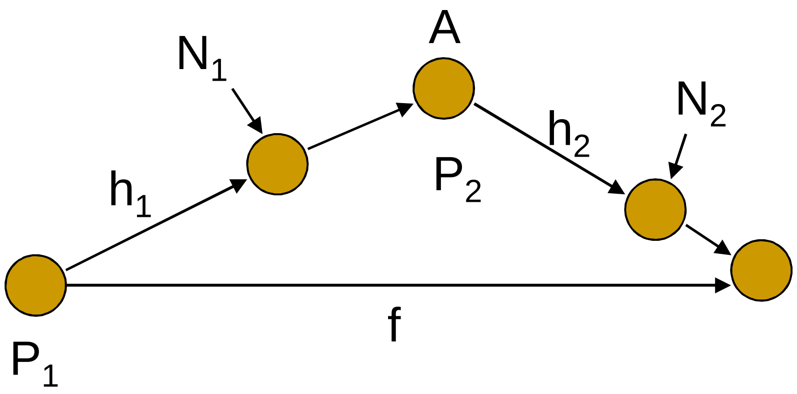
# Performance comparison

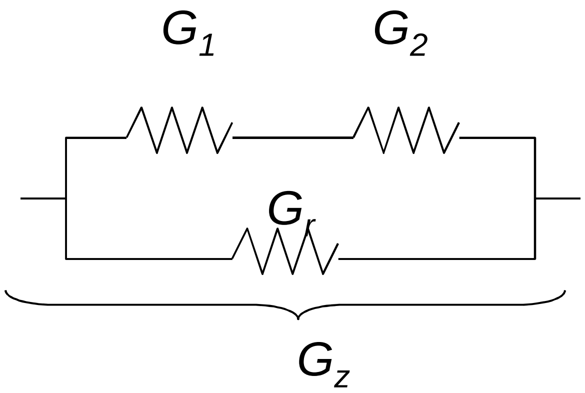


# Performance comparison



# Adding Direct Path





MRC can be used at the receiver. It means that we have two copies of the same signal x, and we use the weighting factors proportional to SNR of each branch. The result is a random variable z with the following SNR:

$$\gamma_z = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} + \gamma_f \quad \text{where} \quad \gamma_f = \frac{P_1 \left| f \right|^2}{N_0}$$

Here the approximation for SNR of relayed signal is used (the one is ignored).

The distribution of $\mathrm{SNR}_f$ is exponential.

# Power allocation

- Since the original signal is received twice, the optimal power allocation is not the same as before. The optimal solution can be found by derivation of the total SNR with respect to beta.

Compare with matlab simulation the three previous cases for this case.

# Symbol Error Rate Calculations

The SER is calculated by averaging over the *pdf* of SNR.

$$P_e = \int_0^\infty AQ(\sqrt{k\gamma})\, p_\gamma(\gamma)d\gamma$$

Note that in AWGN, for many constellations, like MPSK and QAM, the symbol error rate can be calculated as $P_{s-err} = AQ(\sqrt{k\gamma_s})$
Where k depends on constellation.
For example for BPSK, A=1,k=2 ; QPSK: A=2,k=1;

MPSK: $P_{s-err} \approx 2Q\left(\sqrt{2\gamma_s}\sin\left(\frac{\pi}{M}\right)\right)$

Note that the Q function goes to zero very fast with increasing gamma. So the integral has only significant values at small SNRs. It means the $P_e$ is well dimensioned by low SNRs.

# Symbol Error Rate Approximation

$$\int_0^{\infty} Q(\sqrt{k\gamma})\, p_{\gamma}(\gamma)\, d\gamma = \int_0^{\infty} Q(\sqrt{k\overline{\gamma}\hat{\gamma}})\, p_{\hat{\gamma}}(\hat{\gamma})\, d\hat{\gamma}$$

Where $\hat{\gamma} = \gamma / \overline{\gamma}$ has the “same distribution” as $\gamma$ with unity average.

It is clear that the PDF of $\gamma$ around zero is the most important because for larger $\gamma$ the product of the two functions can be ignored.

It is proved[1] that for a system with diversity order t+1, the first term in the Taylor series of $p_{\hat{\gamma}}(\hat{\gamma})$ will be

$$p_{\hat{\gamma}}(\hat{\gamma}) = a\hat{\gamma}^{t} + o(\hat{\gamma})$$

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1- Zhengdao Wang; Giannakis, G.B., "A simple and general parameterization quantifying performance in fading channels," *Communications, IEEE Transactions on* , vol.51, no.8, pp. 1389-1398, Aug. 2003

# Symbol Error Rate Approximation

This approximation can be plugged in the integral and we obtain:

$$\int_{0}^{\infty} Q(\sqrt{k\overline{\gamma}\hat{\gamma}})\, p_{\hat{\gamma}}(\hat{\gamma})\, d\hat{\gamma} \approx \int_{0}^{\infty} \frac{e^{-\frac{k\overline{\gamma}\hat{\gamma}}{2}}}{\sqrt{2\pi}\sqrt{k\overline{\gamma}\hat{\gamma}}}\, a\hat{\gamma}^{t}\, d\hat{\gamma}$$

This was obtained using the approximation for Q function:

$$Q(x) \leq \frac{1}{x\sqrt{2\pi}} e^{-x^{2}/2}$$

And using

$$\Gamma(t+1/2) = \frac{\sqrt{\pi}}{2^{t}} \prod_{l=1}^{t} (2l-1)$$

Changing the variable: $x = \frac{k\overline{\gamma}\hat{\gamma}}{2}$

The error probability is finally:

$$\boxed{P_{s} \leq \frac{Aa}{(k\overline{\gamma})^{t+1}} \prod_{l=1}^{t} (2l-1)}$$

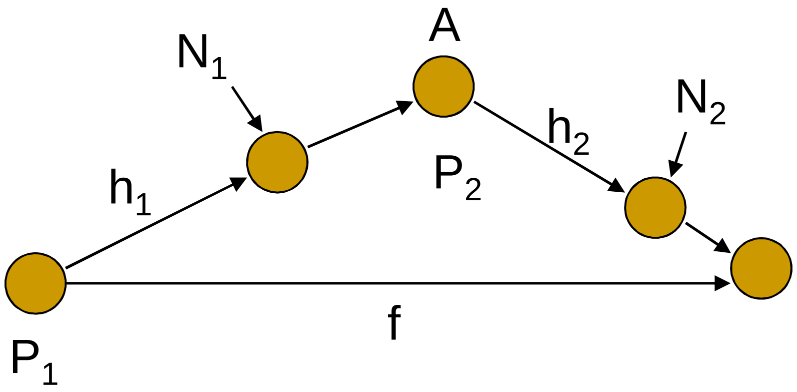
# Taylor Series of $p_\gamma(\gamma)$

As seen before $a = \left.\frac{1}{t!}\frac{\partial^t p_{\hat{\gamma}}(\hat{\gamma})}{\partial \hat{\gamma}^t}\right|_{\hat{\gamma}=0}$

Therefore $$P_s = A\frac{\prod_{l=1}^{t}(2l-1)}{k^{t+1}}\frac{1}{t!}\frac{\partial^t p_\gamma}{\partial \gamma^t}(0)$$

Here the $\hat{\gamma}$ is replaced by $\gamma$ , that's why the $\overline{\gamma}$ at the denominator was simplified.

# Cooperative Configuration

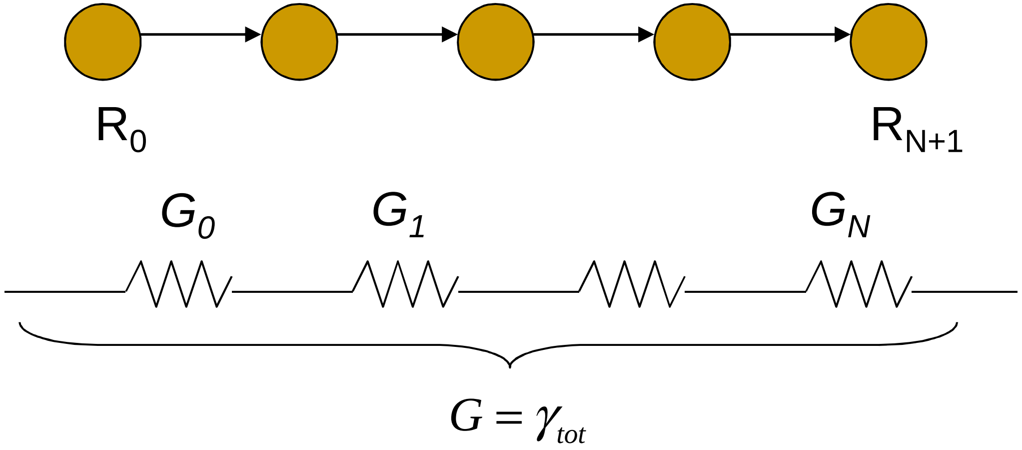


So we should study now the distribution of $\gamma_z = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} + \gamma_f$

$\gamma_z$ is a random variable and $\frac{\partial p_{\gamma_z}}{\partial \gamma_z}(0) = \left( p_{\gamma_1}(0) + p_{\gamma_2}(0) \right) p_{\gamma_f}(0)$

$$P_s = A \frac{\prod_{l=1}^{t} (2l-1)}{k^{t+1}} \frac{1}{t!} \frac{\partial^t p_\gamma}{\partial \gamma^t}(0) = A \frac{1}{k^2} \left( p_{\gamma_1}(0) + p_{\gamma_2}(0) \right) p_{\gamma_f}(0)$$

# Multihop Serial Case



$$\gamma = \frac{\gamma_0 \gamma_1 \cdots \gamma_N}{\sum_{i=0}^{N} \gamma_0 \gamma_1 \cdots \gamma_{i-1} \gamma_{i+1} \cdots \gamma_0}$$

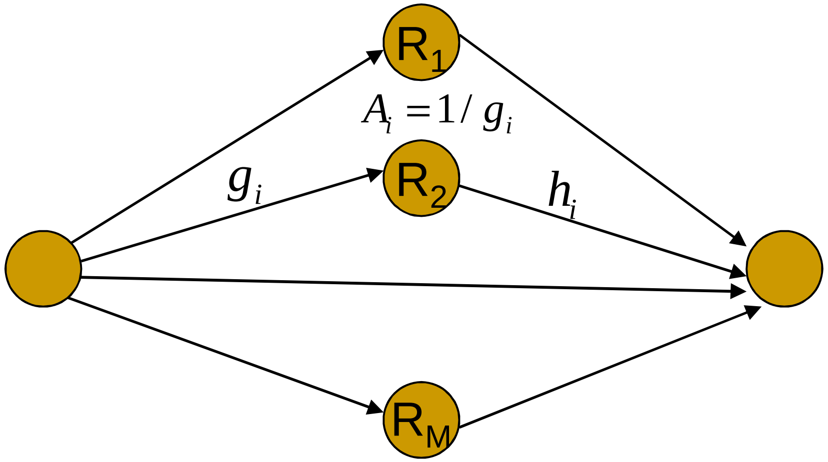
or

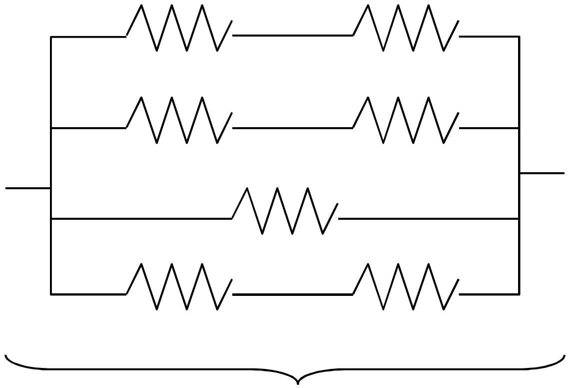
$$\frac{1}{\gamma} = \frac{1}{\gamma_0} + \frac{1}{\gamma_1} + \ldots + \frac{1}{\gamma_N}$$

The error probability can be approximated by [1]: $\bar{P}_e \approx \frac{1}{2k} \sum_{i=0}^{N} P_{\gamma_i}(0)$

1- A.Ribeiro, X. Cai, G. Giannakis, “Symbol Error probabilities for General Cooperative Links”, IEEE trans on commun., May 2005.

# Parallel Two-Hop Relays





$G_{tot} = \gamma_{tot}$

$$\text{MRC} \quad \Rightarrow \quad z = \left(\frac{f}{\sigma_D^2}\right)^* y_D + \sum_{i=1}^{M} \left(\frac{h_i A_i g_i}{\sigma_{R_i}^2}\right)^* y_{R_i}$$

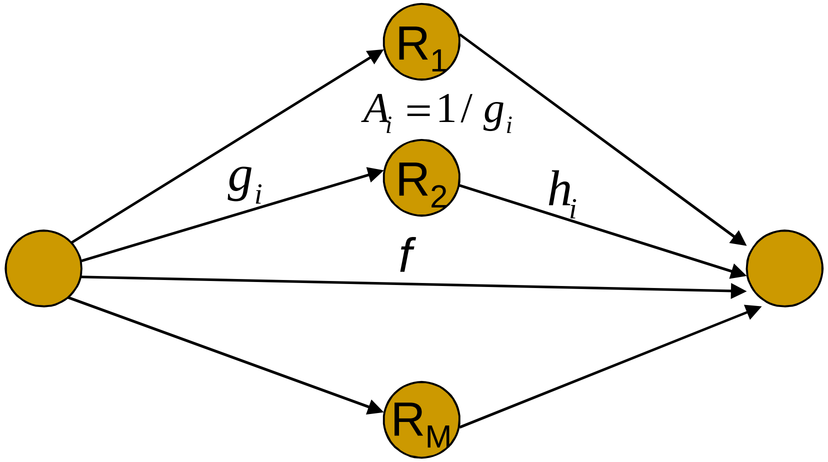
$$\gamma_z = \gamma_f + \sum_{i=1}^{M} \frac{\gamma_{g_i}\gamma_{h_i}}{\gamma_{g_i} + \gamma_{h_i}} \qquad \frac{\partial^M p_{\gamma_z}(\gamma_z = 0)}{\partial \gamma_z{}^M} = p_{\gamma_f}(0) \prod_{i=1}^{M} \left[ p_{\gamma_{g_i}}(0) + p_{\gamma_{h_i}}(0) \right]$$

# Selection AF (S-AF) scheme [1][2]

- In this scheme only the “best relay” is used for relaying
- The best relay is the one who

$$i = \arg\max_i \frac{\frac{|g_i|^2 E_s}{N_{s,i}} \frac{|h_i|^2 E_i}{N_{i,d}}}{\frac{|g_i|^2 E_s}{N_{s,i}} + \frac{|h_i|^2 E_i}{N_{i,d}} + 1}$$

$$= \arg\max_i \frac{\gamma_{i,1}\gamma_{i,2}}{\gamma_{i,1} + \gamma_{i,2} + 1}$$



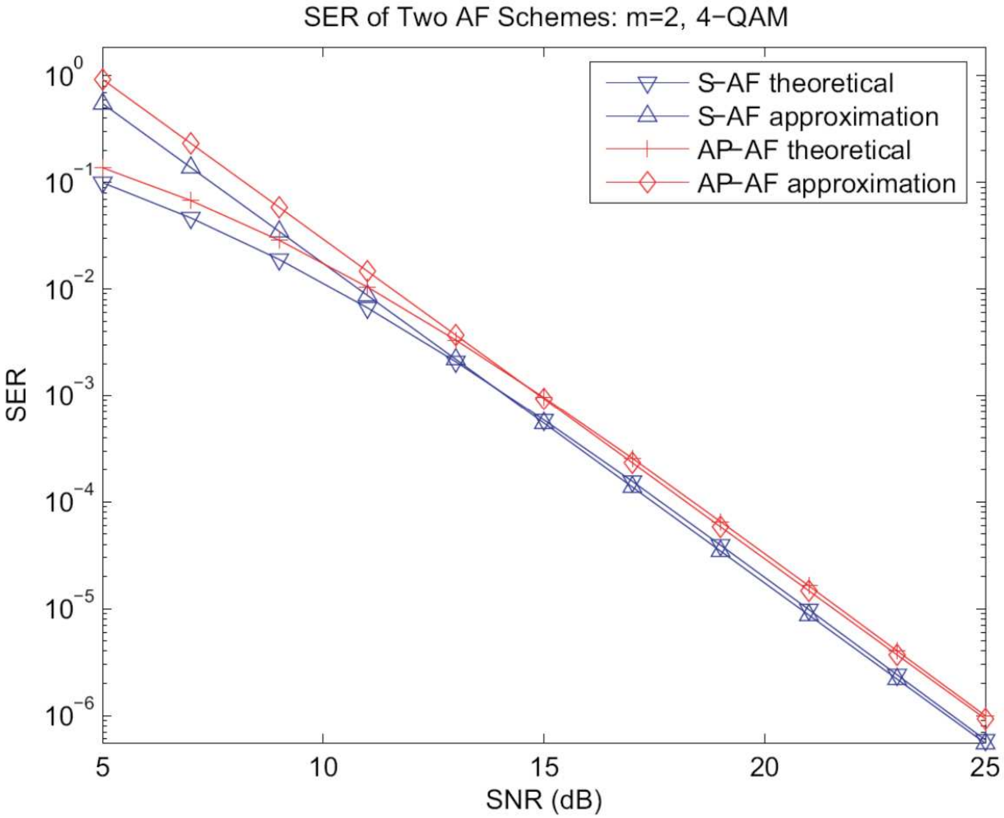
1- Yi Zhao; Adve, R.; Teng Joon Lim, "Symbol error rate of selection amplify-and-forward relay systems," *Communications Letters,* IEEE , vol.10, no.11, pp.757-759, November 2006

2- Zhao, Y.; Adve, R.; Lim, T.J., "Improving amplify-and-forward relay networks: optimal power allocation versus selection," Wireless Communications, IEEE Transactions on , vol.6, no.8, pp.3114-3123, August 2007

# AP-AF versus S-AF

- In S-AF the required parallel (orthogonal) channels are less
  - Just 2 channels are required versus N+1 in AP-AF
- In S-AF the exact knowledge of the channels is not required.
- A full diversity of m+1 is obtained in both cases.
- Because the best channel uses all the power, better performance can be obtained

$$\frac{P_e^{\,S}}{P_e^{\,AP}} = m!\left(\frac{2}{m+1}\right)^{m+1}$$



*m* is the number of relays

# SER calculation of S-AF

$$\gamma_r^S = \alpha_0\gamma + \max_i \frac{\alpha_i\beta_i\gamma^2}{\alpha_i\gamma + \beta_i\gamma + 1}$$

Where $\alpha_0 = |f|^2 E_s,\ \alpha_i = |g_i|^2 E_s,\ \beta_i = |h_i|^2 E_i, \text{ and } \gamma = 1/N_0$

It can be shown that

$$F_{\Gamma_R}^S(\gamma_r) = \frac{\lambda_0 \prod_{i=1}^m (\lambda_i + \xi_i)}{m+1} \left(\frac{\gamma_r}{\gamma}\right)^{m+1}$$

where λ$_0$ is the exponential distribution parameter of α$_0$ : $f_{\alpha_0}(x) = \lambda_0 \exp(-\lambda_0 x)$

And the same for $\lambda_i$ and $\xi_i$

$$P_s \approx \frac{(2m+1)!\lambda_0 \prod_{i=1}^m (\lambda_i + \xi_i)}{(m+1)!(2k\gamma)^{m+1}}$$

So full diversity of *m+1* is achieved.

# Conclusion over AF transmission

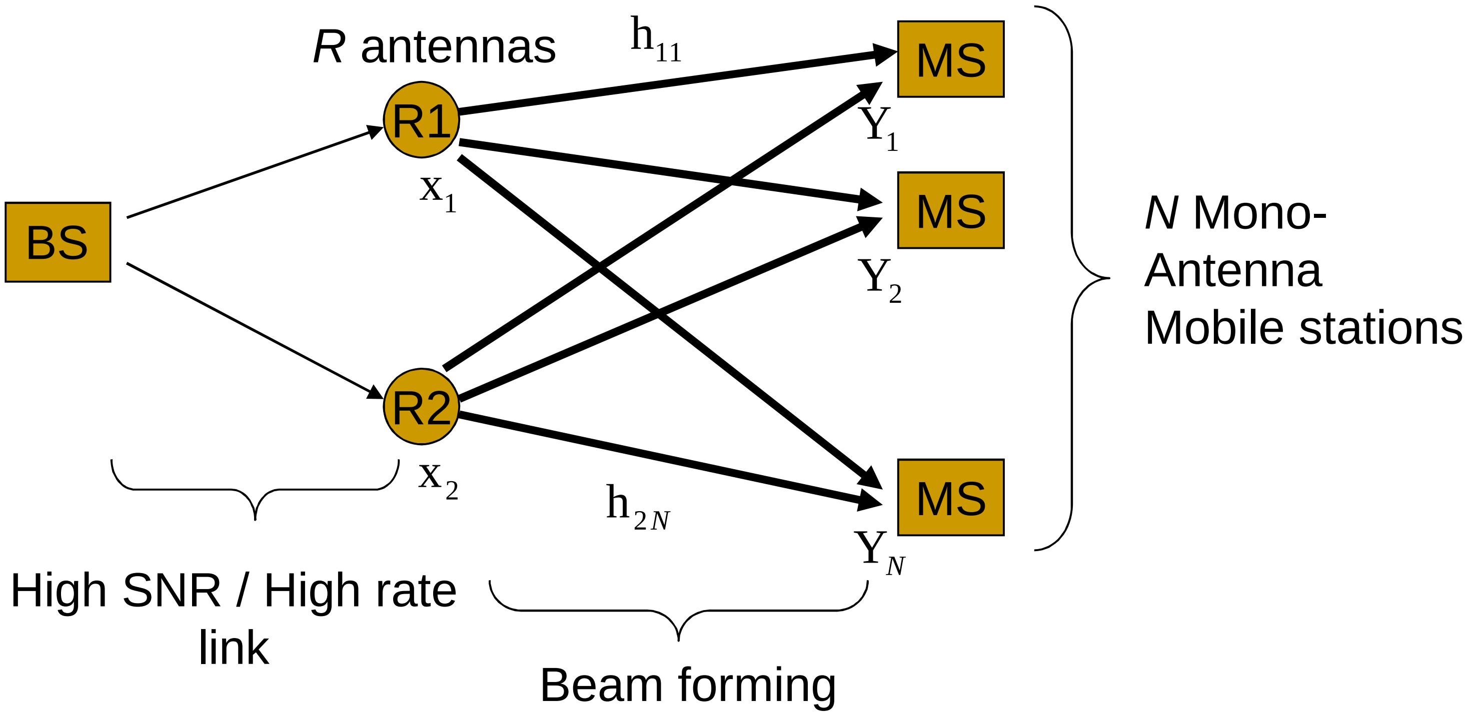
- The outage probability is improved considerably
- Independent channels obtained
- Use of cooperative network is justified in mobile communications
- AF allows full diversity and simple relays
- S-AF give better performance with simpler receiver
- Tight upper bound for high SNR can be analytically obtained

# Plan

- Amplify and Forward (AF) Scheme
  - Two-hop serial and parallel
  - Multi-hop serial and parallel
  - SNR calculations with MRC at the destination
  - SER calculation for high SNR
  - All Participate AF (AP-AP) versus Selection AF (S-AF)
- Cooperative multiple access transmission using precoding vector

# Multiple access cooperative transmission[1]

- One transmitter and several receivers.
- Each receiver is interested on its own data.



H. Meghdadi, V. Meghdadi, J.P. Cances, “Cooperative Multiple Access Transmission Using Precoding vectors” EUSIPCO 2009.

# Problem

- Cancelling multi-antenna and multi-relay interference
- Maximizing the SNR at destinations
- This is done using linear filtering: weighting vector at relay nodes
- The BS send a vector of symbols of size $N$.
- Each relay, by one channel use, send a sequence of size $R$ over its $R$ antennas.
- Assumption
  - Each relay knows all the channels (CSI)
  - Each relay only know its own channel

# System model

Transmitted information symbols $\mathbf{s} = \left[s_1, s_2, \cdots s_N\right]^T$

Signals at the output of the relays (of size *R* where *R* is the number of TX antennas)

$$\mathbf{x}_1 = \sum_{j=1}^{N} s_j \mathbf{w}_j^1 \qquad \mathbf{x}_2 = \sum_{j=1}^{N} s_j \mathbf{w}_j^2$$

Signal at the *j* th mobile.

$$\begin{aligned} y_j &= \mathbf{h}_{1j} \cdot \mathbf{x}_1 + \mathbf{h}_{2j} \cdot \mathbf{x}_2 + n_j \qquad j = 1, \ldots, N \\ &= \mathbf{h}_{1j} \cdot \sum_{k=1}^{N} s_k \mathbf{w}_k^1 + \mathbf{h}_{2j} \cdot \sum_{k=1}^{N} s_k \mathbf{w}_k^2 + n_j \end{aligned}$$

$\mathrm{W}_j^i$ are the precoding vectors to be optimized.

# Interference cancellation

All the contributions of all other symbols except for *j* th should be cancelled out.

First case: each relay knows all the CSI

$$\sum_{k\neq j} s_k \mathbf{h}_{1j} \cdot \mathbf{w}_k^1 + \sum_{k\neq j} s_k \mathbf{h}_{2j} \cdot \mathbf{w}_k^2 = 0$$

Second case: each relay knows only its own channel

$$\sum_{k\neq j} s_k \mathbf{h}_{1j} \cdot \mathbf{w}_k^1 = 0 \quad , \qquad \sum_{k\neq j} s_k \mathbf{h}_{2j} \cdot \mathbf{w}_k^2 = 0$$

Therefore:

$$y_j = s_j \underbrace{\left(\mathbf{h}_{1j} \cdot \mathbf{w}_j^1 + \mathbf{h}_{2j} \cdot \mathbf{w}_j^2\right)}_{\text{Must be a real positive number}} + n_j$$

# Precoding vector calculations

Definitions:

$$\mathbf{H}_i = \left[\ \mathbf{h}_1^{i\ T} \mid \mathbf{h}_2^{i\ T} \mid \ldots \mid \mathbf{h}_N^{i\ T}\ \right]^T_{R\times N} \qquad , i = 1, 2$$

$$\mathbf{W}_i = \left[\ \mathbf{w}_1^{i} \mid \mathbf{w}_2^{i} \mid \ldots \mid \mathbf{w}_N^{i}\ \right]_{R\times N} \qquad , i = 1, 2$$

$$\mathcal{W}_i = \left[\ \mathbf{w}_1^{i\ T} \mid \mathbf{w}_2^{i\ T} \mid \ldots \mid \mathbf{w}_N^{i\ T}\ \right]^T_{NR\times 1} \qquad , i = 1, 2$$

Using these notations, the second case interference cancelation becomes

$$\mathbf{A}_{i1}\mathcal{W}_i = \mathbf{0} \quad \text{with } \mathbf{A}_{i1} = \left(\mathbf{s}^T \otimes \mathbf{1}_{N\times 1} - \text{diag}(\mathbf{s})\right) * \mathbf{H}_i$$

Kronecker product

Row wise Kronecker product

# Transforming on real equation

The equation $\mathbf{A}_{i1}\boldsymbol{\mathcal{W}}_i = \mathbf{0}$ can be written as

$$\Re\left\{\left(\mathbf{A}_{i1} \otimes \begin{bmatrix} 1 & j \\ -j & 1 \end{bmatrix}\right)\right\} \Re\left\{\left(\boldsymbol{\mathcal{W}}_i \otimes \begin{bmatrix} 1 \\ -j \end{bmatrix}\right)\right\} = \mathbf{0}_{2N\times 1}$$

Therefore we have $\boxed{\hat{\mathbf{A}}_{i1}\hat{\boldsymbol{\mathcal{W}}}_i = \mathbf{0}_{2N\times 1}}$ with real entries.

# Coherent addition

Respecting this constraint, the received signal at each MS will be:

$$y_j = s_j\left(\mathbf{h}_{1j}\cdot\mathbf{w}_j^1 + \mathbf{h}_{2j}\cdot\mathbf{w}_j^2\right) + n_j$$

The coefficient of $s_j$ should be a positive real, so :

$$\Im\left\{\mathbf{h}_{ij}\cdot\mathbf{w}_j^i\right\} = 0, \qquad i = 1, 2, \quad j = 1\cdots N$$

Using matrix representation:

$$\Im\left\{\mathbf{A}_{i2}\mathcal{W}_i\right\} = \Im\left\{\left(\mathbf{I}_N * \mathbf{H}_i\right)\mathcal{W}_i\right\} = 0, \qquad i = 1, 2$$

$$\Re\left\{\left(\mathbf{A}_{i2}\otimes\begin{bmatrix}-j & 1\end{bmatrix}\right)\right\}\Re\left\{\left(\mathcal{W}_i\otimes\begin{bmatrix}1\\-j\end{bmatrix}\right)\right\} = \mathbf{0}_{N\times 1} \quad\Rightarrow\quad \boxed{\hat{\mathbf{A}}_{i2}\hat{\mathcal{W}}_i = \mathbf{0}_{N\times 1}}$$

# Power constraint and objective function

This is the optimization constraint which limits the transmitted power of each relay. $\mathcal{W}_i^H \mathcal{W}_i \leq P_i$

Therefore the function to be maximized is the SNR at mobile stations with the following constraints

$$\hat{\mathbf{A}}_{i1}\hat{\mathcal{W}}_i = \mathbf{0}_{2N\times 1} \qquad \hat{\mathbf{A}}_{i2}\hat{\mathcal{W}}_i = \mathbf{0}_{N\times 1} \qquad \mathcal{W}_i^H \mathcal{W}_i \leq P_i$$

Instead, we can minimize the power but fix the SNR. This can be better optimized using Lagrange multipliers.

So the function to be minimized is: $\hat{\mathcal{W}}_i^T \hat{\mathcal{W}}_i$

Note: We are looking for the solutions independent of **s**.

# SNR constraint

- One possibility is to maximize the sum of the all the SNRs at all MS.
- Another possibility is to make MS’s SNR equal and to maximize it
- Another possibility can be to let the SNRs to be different. This will let us to have the MS with more privilege.

# Solution

- Case: flexible constraints over SNR at MSs
  - Solution : Lagrange multipliers
- Special case : SNRs at all MS are forced to be equal
  - Solution : Lagrange multipliers method gives the same answer as pseudo inverse method

$$\mathbf{y} = (\mathbf{H}_1\mathbf{W}_1 + \mathbf{H}_2\mathbf{W}_2)\,\mathbf{s} + \mathbf{n}$$

$$= \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 \end{bmatrix} \begin{bmatrix} \mathbf{W}_1 \\ \mathbf{W}_2 \end{bmatrix} \mathbf{s} + \mathbf{n} = \mathbf{H}_{N\times 2R}\mathbf{W}_{2R\times N}\mathbf{s} + \mathbf{n}.$$

So we require $\mathbf{HWs} = g\mathbf{s}$. Normalizing by using *g*=1:

$$\mathbf{H}_{N\times 2R}\mathbf{W}'_{2R\times N} = \mathbf{I}_N$$

# Solution …

- The solution is $\mathbf{W}' = \mathbf{H}^H \left(\mathbf{H}\mathbf{H}^H\right)^{-1}$

Note, H must be full rank : $R \geq N/2$

- In this solution, all the relays had to know all the CSI. If each relay knows only its own channel to MSs :

$$\mathbf{W}'_i = \mathbf{H}_i^H \left(\mathbf{H}_i\mathbf{H}_i^H\right)^{-1} \qquad , i = 1, 2$$

Note, $H_i$ must be full rank : $R \geq N$

- De-normalizing

$$\mathbf{W}_i = \frac{\sqrt{P_i}\,\mathbf{W}'_i}{\|\mathbf{W}'_i\|}$$

# Performance

- The received signal is (a vector of size *N*)

$$y_j = \frac{\sqrt{P}}{\|\mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H\right)^{-1}\|} s_j + n_j$$

- SNR will be

$$\gamma = \frac{P}{N_0\|\mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H\right)^{-1}\|^2}$$

$$\gamma = \frac{P}{N_0} \cdot \frac{1}{\mathrm{trace}\left((\mathbf{H}\mathbf{H}^H)^{-1}\right)}$$

- It is difficult to obtain the distribution of γ, so we calculate an upper bound for BEP for high SNR.

# BEP calculations

- In general, the symbol error probability can be calculated as:

$$P_s = \int_0^\infty 2Q(\sqrt{k\beta\bar{\gamma}})P_\beta(\beta)d\beta$$

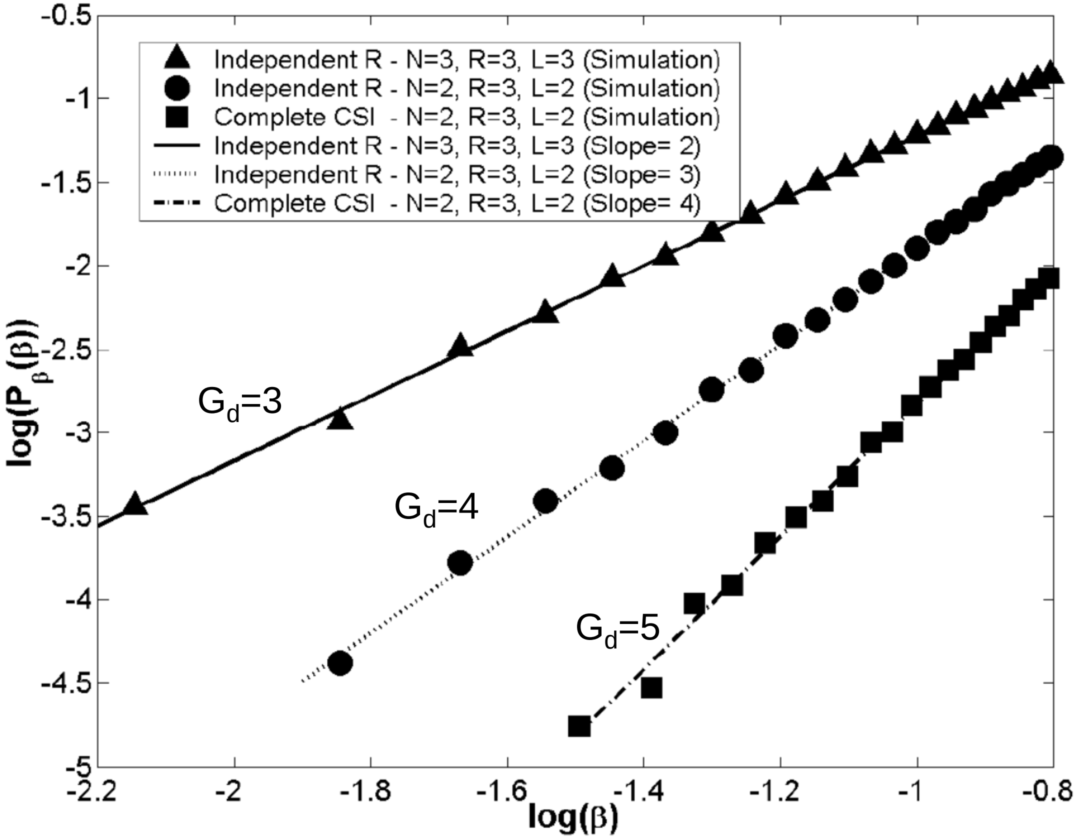
where $\beta = \gamma/\bar{\gamma}$

- Replacing $Q(x) \leq \frac{1}{x\sqrt{2\pi}}e^{-\frac{x^2}{2}}$ and using Taylor series

for $P_\beta(\beta)$ and supposing a diversity order of *t*+1

$$P_s \approx \int_0^\infty 2\frac{1}{\sqrt{2\pi k\beta\bar{\gamma}}}e^{-\frac{k\beta\bar{\gamma}}{2}}a\beta^t d\beta = \frac{2a}{(k\bar{\gamma})^{t+1}}\prod_{l=1}^{t}(2l-1)$$

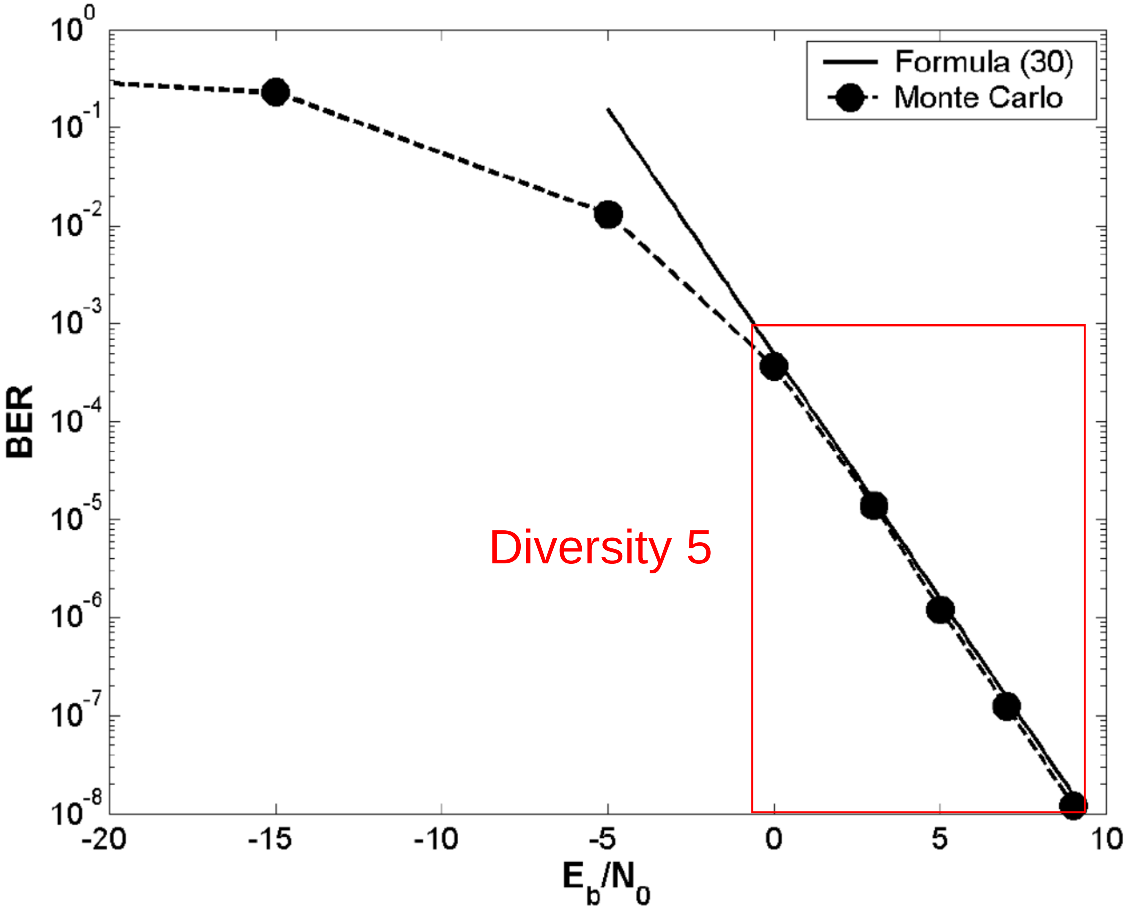
# Simulation



Independent relays: $G_d = LR - L(N-1)$

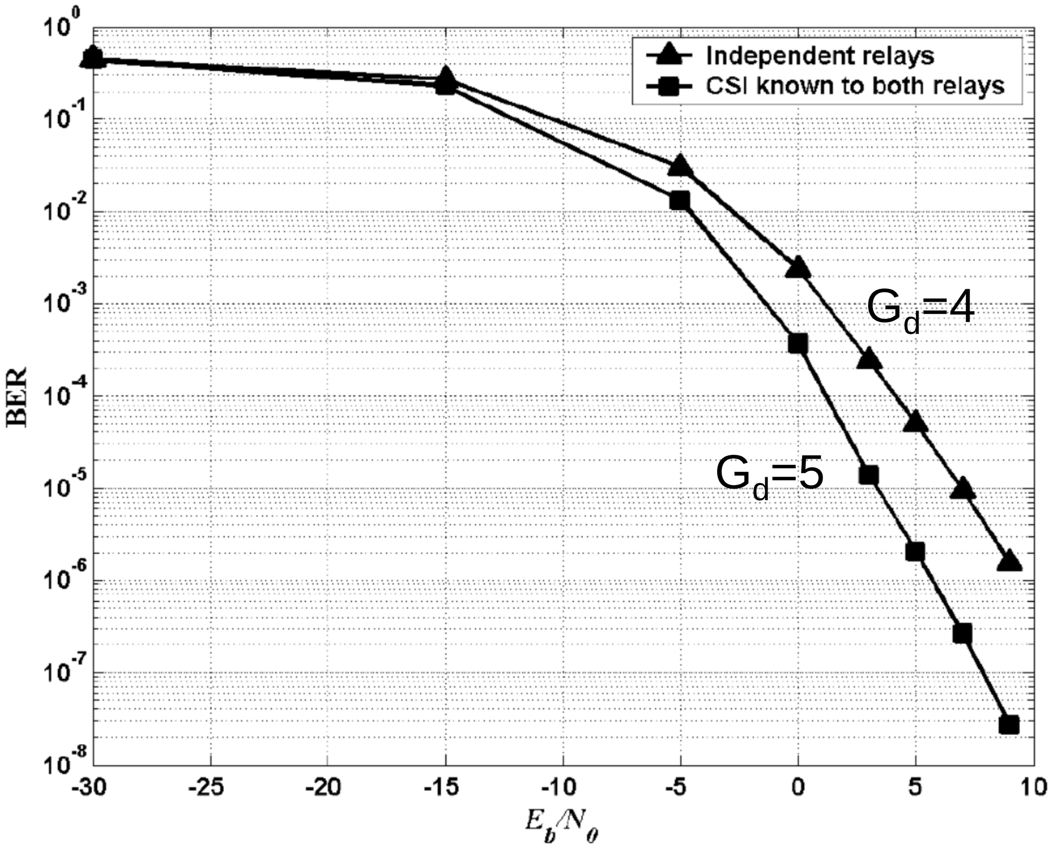
Complete CSI : $G_d = LR - (N-1)$

# Simulation



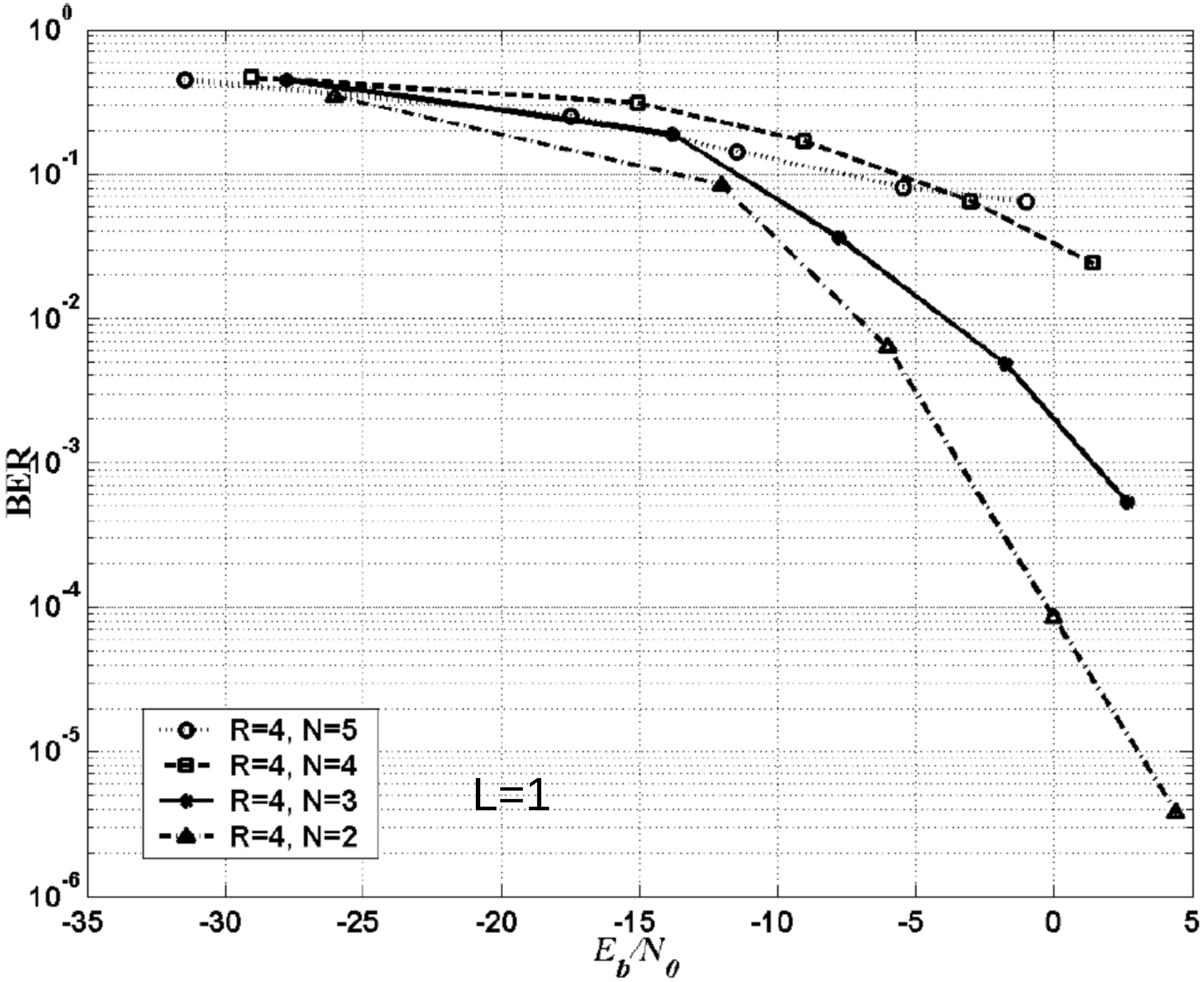
Complete CSI and *L*=2, *N*=2, and *R*=3

# Simulation



$L$=2, $N$=2, and $R$=3

# Simulation



L=1

# Conclusion

- Optimum precoding vectors calculated for different scenarios, thanks to Lagrange multipliers.
- The special case where the constraint that all the SNR at the mobile stations are the same reduces to Moore-Penrose pseudo-inverse.
- The case where all relays knows all the CSI is considered which gives a single large matrix pseudo-inversion.
- The case where each mobile knows only its own channel is also considered which is less performing but easier to implement practically.
- Semi analytic tight upper bound is evaluated for general case.