STh-01

MODERN ADVANCES IN COMPUTATIONAL IMAGING AT MICROWAVE AND MILLIMETRE-WAVE FREQUENCIES

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New advances in computational imaging: Polarimetric imaging, phaseless imaging, and recent advances in antenna systems and k-space reconstruction techniques

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Outlines of the second part

• Acceleration of image reconstruction: k-space and Fourier processing
• Computational polarimetric imaging
• Computational phaseless imaging
Accelerating computational imaging with Fourier processing

Some problems require the estimation of millions of voxels

Using MIMO apertures, the number of measured samples can also reach very large numbers

Sensing matrix

Transmitters \times Receivers \times Frequency samples

Example

Transmitters : 40 \times 40
Receivers : 40 \times 40
Frequency points : 20

\begin{align*}
 nbx & : 300 \\
nby & : 200 \\
nbz & : 300
\end{align*}

\text{size}(H) = 51.10^6 \times 18.10^6

\text{Raw memory consumption} \sim 2200 \text{Tb (single precision)}
What makes Fourier processing so efficient?

Propagation is modelled within a scalar model: waves progress from source points in a spherical manner.

\[ G(r, r_a) = \frac{e^{-jk|r-r_a|}}{|r-r_a|} \]

**Note:** Explanations are given for a set of 2D examples. For the sake of simplicity, we use an asymptotic form of Hankel's functions:

\[ H_0^{(2)}(kr) \approx \frac{e^{-jkr}}{\sqrt{r}} \]
What makes Fourier processing so efficient?

Following an angular spectrum decomposition, a spherical wave is analogous to a sum of plane waves.

Weyl's expansion:

\[ \frac{e^{jkr}}{r} = \frac{j}{2\pi} \int \int e^{j(k_xx + k_z z +|k_y|y)} \, dk_x \, dk_z \]

with: \[ k_y = \sqrt{k^2 - k_x^2 - k_z^2} \]
What makes Fourier processing so efficient?

Impact of a spatial Fourier transform on measurements:

$$S(k_x, f) = \mathcal{F}(S(x, f))$$

The transformation helps to identify a set of angles of arrival.
What makes Fourier processing so efficient?

Impact of a spatial Fourier transform on measurements:

\[ S(k_x, f) = \mathcal{F}(S(x, f)) \]

The transformation helps to identify a set of angles of arrival.
What makes Fourier processing so efficient?

A fundamental concept for short-range Fourier imaging: the definition of a dispersion relation

An association of transmitted and received plane waves are considered:

\[ k_y = k_t + k_r \]

**In the SIMO case:**
A single Tx plane wave interacts with a set of Rx plane waves

\[ k_y = k \sin(\theta_t) + \sqrt{k^2 - k_x^2} \]

We choose an area to image whose center is called the **stationary phase point**.

---

*Product of Tx and Rx Green’s functions: sum of the arguments of their exponential functions (the rigorous demonstration is not that straightforward)*
What makes Fourier processing so efficient?

Full process for image computation:

1. Fourier transform and variable change

\[ S(x, f) \rightarrow S(k_x, f) \rightarrow C S(k_x, k) \]

2. Back-propagation

\[ S(k_x, y) = \int S(k_x, k) e^{jk_y} dk_x \]

3. Inverse Fourier transform

\[ I(x, y) = \mathcal{F}^{-1}(S(k_x, y)) \]

This approach is often referred as the Range Migration Algorithm (RMA) or k-ω algorithm


Note:
1. The back-propagation step can be greatly accelerated using Stolt interpolation instead of a summation.
2. In this example, we add an additional phase shift \( e^{jk_x y} \) to compensate for the offset of the transmit antenna.
Making Fourier techniques compatible with computational imaging

**Problem:** the use of frequency-diverse antenna does not grant us direct access to the spatial dimension

Measurement of a "compressed" signal:

\[ \rho_\omega = \int_{r_t} \int_{r} G_\omega(r_t, r) f(r) G_\omega(r, r_r) \, d^3r \, H_\omega(r_r) \, d^2r_r \]

Identification of signals in the radiating aperture

\[ s_\omega(r_r) = \int_{r} G_\omega(r_t, r) f(r) G_\omega(r, r_r) \, d^3r \]

How to efficiently retrieve the signals in the aperture?
Making Fourier techniques compatible with computational imaging

**Problem:** the use of frequency-diverse antenna does not grant us direct access to the spatial dimension

![Diagram](image)

Measurement of a "compressed" signal

\[
\rho_\omega = \int_{r_r} \int_{r_t} G_\omega(r_t, r) f(r) G_\omega(r, r_r) \, d^3r \, d^2r_r
\]

Identification of signals in the radiating aperture

\[
s_\omega(r_r) = \int_r G_\omega(r_t, r) f(r) G_\omega(r, r_r) \, d^3r
\]

The signals undergo a known distortion

\[
\rho_\omega = \int_{r_r} s_\omega(r_r) \, H_\omega(r_r) \, d^2r_r
\]

In matrix form

\[
\rho_\omega = H_\omega^T s_\omega \quad \text{with} \quad H_\omega \in \mathbb{C}^{n_{r_r} \times 1} \quad s_\omega \in \mathbb{C}^{n_{r_r} \times 1}
\]

**Problem:** a simple matrix inversion reconstruction requires the sacrifice of a dimension

\[
\hat{s}_o = H^+ \rho \quad \text{with} \quad H \in \mathbb{C}^{n_f \times n_{r_r}} \quad \rho \in \mathbb{C}^{n_f \times 1} \quad s_o \in \mathbb{C}^{n_{r_r} \times 1}
\]

**Pros:** we just retrieved our lost spatial dimension

**Cons:** our frequency dimension is now missing
Making Fourier techniques compatible with computational imaging

**Problem:** the use of frequency-diverse antenna does not grant us direct access to the spatial dimension

An estimation is carried out using an equalization

\[ \hat{S}_{eq} = H^+ \odot (\rho, \ldots, \rho) \]

The measured "compressed" signal is multiplied by each line of the pseudo-inverse matrix.

The signals reconstructed in the radiating aperture can finally be used to reconstruct images using Fourier techniques.

The measurement can also be written as a **Hadamard product** (element-wise multiplication)

\[ \rho = \sum_{r_T} H \odot S \quad \text{with} \quad H \in \mathbb{C}^{n_f \times n_T T}, \quad S \in \mathbb{C}^{n_f \times n_T r} \]
Performance comparison: Equalization+RMA VS full-matrix approach

<table>
<thead>
<tr>
<th></th>
<th>Pre-computation time</th>
<th>Computation time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equalization+RMA</td>
<td>4.8 s</td>
<td>2.4 s</td>
</tr>
<tr>
<td>Full-matrix</td>
<td>38 min</td>
<td>66 s</td>
</tr>
</tbody>
</table>

*Data presented in 2015, significant progress has been made on both methods since*
Examples of implementations

From 4 antennas to 1 receiver

Frequency range: 2-4 GHz
7 fps (limited by communication times between equipment)
Examples of implementations

**From 16 antennas to 4 receivers**
Frequency range: 2-4 GHz

Improvement of the equalization technique by a sparsity-driven technique based on Toeplitz matrices
Examples of implementations

Going MIMO:
From 24x24 antennas to 1 transceiver

Frequency range: 2-10 GHz

Image computation:
1.5 s with MIMO RMA

Improving the MIMO RMA

Bottleneck of the RMA:
interpolating the Tx and Rx plane waves

\[ k_{x} = k'_{xT} + k'_{xR} \]
\[ k_{z} = k'_{zT} + k'_{zR} \]
\[ k_{y} = \sqrt{k'_{xT}^2 - k'_{xR}^2 - k'_{zT}^2} + \sqrt{k'_{xR}^2 - k'_{zR}^2 - k_{xT}^2} \]

Plane wave fusion can be computationally intensive

Everything else can be done from Fast-Fourier transforms

\[ \hat{\sigma}(x, y, z) = k_{yT} k_{yR} \int_{k_{xT}} \int_{k_{yT}} \int_{k_{z}} S(k_{xT}, k_{zT}, k_{xR}, k_{zR}, k) e^{i k_{y} y} e^{i k_{z} z} dk_{x} dk_{y} dk_{z} \]
Improving the MIMO RMA

The initial approach is quite abstract and based on asymptotic developments of radiation integrals.

To the authors’ knowledge, all asymptotic developments proposed in this field are based on a simplified derivation of the measured signals, considering that amplitude terms have negligible influence in comparison with phase terms and can thus be removed. It is shown in this section that it is necessary to keep the decay term 1/\( R_z \) evaluated at the stationary phase point so that the final expression converges towards a form very close to that given in the references mentioned above. From a physical point of view, the conservation of the amplitude term also seems justified as far as this information remains of particular interest in the context of imaging applications in the Fresnel zone.

We start the calculation from the initial formulation of the MIMO signal simplified according to a scalar field approximation and to first-order Born approximation:

\[
s(z_1, z_2, z_1, z_2, k) = \int \int \frac{\phi(x,z_1) e^{-i\kappa z_1} e^{-i\kappa z_2}}{4 \pi^2 R_1 R_2} d\theta_1 d\theta_2 d\Phi_1 d\Phi_2
\]

with

\[
R_1 = \sqrt{(x-x_1)^2 + (z_1-z_1)^2}
\]

\[
R_2 = \sqrt{(x-x_2)^2 + (z_2-z_2)^2}
\]

\[
\Phi = -\sqrt{(x-x_1)^2 + (z_1-z_1)^2} + \frac{\kappa_1}{k} (x_1 - x) + \frac{\kappa_1}{k} (z_1 - z)
\]

\[
R = \sqrt{(x-x_2)^2 + (z_2-z_2)^2}
\]

The spatial dimensions are expressed in the Fourier domain in order to consider the interference between the emitted and received plane waves to be imagined.

\[
S(z_1, z_2, z_1, z_2, k) = \mathcal{B} \left( \phi(s, z_1, z_2, z_1, z_2) \right)
\]

The development of the expression of this signal makes it possible to factorize the transmission and reception terms.

\[
S = \int \int \frac{\phi(x,z_1) e^{-i\kappa z_1} e^{-i\kappa z_2}}{4 \pi^2 R_1 R_2} d\theta_1 d\theta_2 d\Phi_1 d\Phi_2
\]

\[
\frac{\phi(x,z_1) e^{-i\kappa z_1} e^{-i\kappa z_2}}{4 \pi^2 R_1 R_2} d\theta_1 d\theta_2 d\Phi_1 d\Phi_2
\]

where the surface elements of the transmit and receive apertures are respectively \( d\theta_1 = d\sigma_1 d\theta_1 \) and \( d\theta_2 = d\sigma_2 d\theta_2 \). The integrals \( E_1 \) and \( E_2 \) share the same mathematical form that can be simplified using the method of stationary phase.

These expressions are developed here with a generic index \( i \) standing for \( 1 \) or \( 2 \):

\[
\Phi = \Phi(x,z_2) + \frac{\partial \Phi}{\partial \theta_1} \Phi_1(x,z_1) + \frac{\partial \Phi}{\partial \theta_2} \Phi_2(x,z_2)
\]

\[
= \Phi(x,z_2) - \frac{\kappa_1}{k} (x_2 - x) - \frac{\kappa_1}{k} (z_2 - z)
\]

where \( \kappa_1 \) is the wavenumber.

The expression of \( \phi(x,z_2) \) can first be obtained from the first derivatives vanishing at the stationary phase point:

\[
\frac{\partial \phi}{\partial \theta_1} = \frac{\partial \phi}{\partial \theta_2} = 0
\]

\[
\Rightarrow (x_2 - x)^2 + (z_2 - z)^2 = \frac{\kappa_1^2}{k^2}
\]

Finally:

\[
\phi(x,z_2) = \frac{\kappa_1}{k} (z_2 - z)
\]

The resolution of this last equation system makes it possible to determine the expression of the coordinates of the stationary phase point, extracting the positive roots for each case:

\[
x_2 = x + \frac{\kappa_1}{k} (z_2 - z)
\]

\[
z_2 = z + \frac{\kappa_1}{k} (x_2 - x)
\]

Finally, it is necessary to evaluate the expression of the phase term at the stationary point \( \Phi(x_2, z_2) \), as well as the expression of the phase term at the stationary point \( \Phi(z_2, z_2) \).

\[
\Phi(x_2, z_2) = \frac{\kappa_1}{k} (x_2 - x)
\]

\[
\Phi(z_2, z_2) = \frac{\kappa_1}{k} (z_2 - z)
\]

The magnitude and phase term evaluated at the stationary point can then be extracted from the integral:

\[
E_i = \frac{\phi(x_2, z_2)}{\Phi(x_2, z_2)} I_p
\]

where \( I_p \) is a Gaussian integral evaluated around the stationary phase point [7, 7].

The last derivative term \( \frac{\partial \phi}{\partial \theta_1} \) is finally evaluated:

\[
\frac{\partial \phi}{\partial \theta_1} = \frac{\partial \phi}{\partial \theta_1} = \frac{\kappa_1}{k} (z_2 - z)
\]

\[
\Phi(x_2, z_2) = \frac{\kappa_1}{k} (z_2 - z)
\]

These three derivatives are finally evaluated at the stationary phase point:

\[
\frac{\partial \phi}{\partial \theta_1} = \frac{\kappa_1}{k} (z_2 - z)
\]

\[
\frac{\partial \phi}{\partial \theta_2} = \frac{\kappa_1}{k} (x_2 - x)
\]

Having \( \alpha > \gamma^2 \) and \( \eta < 0 \), Eq. (38) then takes the following form:

\[
\frac{\partial \phi}{\partial \theta_1} = \frac{\kappa_1}{k} (z_2 - z)
\]

\[
\frac{\partial \phi}{\partial \theta_2} = \frac{\kappa_1}{k} (x_2 - x)
\]
Improving the MIMO RMA

Introducing the transverse spectrum deconvolution (TSD RMA)

\[ S(x_T, z_T, x_R, z_R, f) \]

4D FFT

\[ S(k_{x_T}, k_{z_T}, k_{x_R}, k_{z_R}, f) \]

TSD MIMO RMA

Core idea: it's all about radiation patterns

Main advantages:
- more intuitive and visual approach
- \( k \)-space patterns depend only on the spatial sampling not on the frequency

Improving the MIMO RMA

Core idea: it's all about radiation patterns

Interrogation of a spectrum through known radiation patterns

$$ S = \int_{k_x} \int_{k_z} P_t(\mathbf{k}_x - \mathbf{k}_x', \mathbf{k}_z - \mathbf{k}_z') S_c(k_x, k_z, \mathbf{k}) P_r(\mathbf{k}_x - \mathbf{k}_x', \mathbf{k}_z - \mathbf{k}_z') \, dk_x \, dk_z $$

Expression in matrix form and separation of spatial variables

$$ S(k) = P_x \, S_c(k) \, P_z $$

Reconstruction of transverse spectral data by pseudo-inversion

$$ \hat{S}_c(k) = P_x^+ \, S(k) \, P_z^+ $$

Main advantages:
- more intuitive and visual approach
- $k$-space patterns depend only on the spatial sampling not on the frequency

1. Interpolation calculations are replaced by matrix multiplication
2. These calculations are made without any frequency dependence
3. RAM consumption is greatly reduced
4. Reconstructions are more accurate
Improving the MIMO RMA

Definition of a new state of the art of range-migration algorithms

Application to large and densely populated MIMO arrays

Validation with a MIMO array of 714 radiating elements more than 25 million interactions at each frequency

Polarimetric computational imaging

**Motivation:** Study of anisotropic/multiple-bounce scattering

**Far-field:** Estimation of soil parameters

**Short-range:** Threat detection

Unsupervised Terrain Classification Preserving Polarimetric Scattering Characteristics

Holographic arrays for multi-path imaging artifact reduction
Polarimetric computational localization

Conventional approach

Single polarized antenna

Computational approach

Doubles the hardware constraints compared to scalar systems

Polarimetric computational localization

We now get out of the scalar approximation

\[ g = \int_{r_a} \int_{r} \overline{C}(r_a) \overline{G}(r, r_a) \overline{s}(r) \, dr_a \, dr \]

in the polarization space:

\[
\begin{align*}
\overline{s}(r) & \quad \text{Polarized sources} \\
\overline{C}(r_a) & \quad \text{Transfer function of the cavity (3 polarizations on its surface)} \\
\overline{G}(r, r_a) & \quad \text{Dyadic Green's function}
\end{align*}
\]

To lighten the notation, frequencies are implicit

\[
\overline{G}(r, r_a) = (1 + \frac{1}{k^2 \Delta \Delta} \cdot \exp(-jkr - r_a)) \frac{1}{|r - r_a|}
\]
Polarimetric computational localization

We now get out of the scalar approximation

\[ g = \int_{r_a} \int_{r} \overline{C}(r_a) \overline{G}(r, r_a) \overline{s}(r) \, dr_a \, dr \]

Discrete model

\[ g = \overline{C} \overline{G} \overline{s} \]

Reconstruction

\[ \hat{s} = \left( \overline{G} \overline{C} \right)^+ g \]

To lighten the notation, frequencies are implicit
Polarimetric computational localization

Near field scans $\overline{C}(r_a)$  

Cavity quality factor $Q \approx 12\,000$

Polarimetric computational imaging

Near field scans \( C(r_x) \)

Spatial correlations:

Port 1 - Pol. x

Port 1 - Pol. z

Port 1 - Pol. x

Port 1 - Pol. z

Polarimetric computational localization

Reconstruction

\[ \hat{s} = \left( \overline{G} \overline{C} \right)^{+} g \]

- Frequency range: 18.5-26 GHz
- 801 Frequency samples
- \(134 \times 36 \times 134 \sim 650,000\) voxels
- Volume of \(0.8 \times 0.75 \times 0.8\ \text{m}^3\)
- Computed in less than a second using Fourier processing

-5dB isosurfaces
Polarimetric computational imaging

Scalar imaging forward model

\[ g_{i,j}(\omega) \propto \int_{r} [E_i \cdot E_j](r, \omega) f(r) \, d^3r \]

- \( g_{i,j}(\omega) \): measured frequency-domain signal
- \( E_i(r, \omega) \): scalar Tx electric field
- \( E_j(r, \omega) \): scalar Rx electric field
- \( f(r) \): scalar reflectivity function (\( \Delta \varepsilon(r) \))

If our target exhibits anisotropic/depolarizing behavior:

Each point in space must be defined as a susceptibility tensor

\[
\chi = \begin{bmatrix}
\chi_{xx} & \chi_{xy} & \chi_{xz} \\
\chi_{yx} & \chi_{yy} & \chi_{yz} \\
\chi_{zx} & \chi_{zy} & \chi_{zz}
\end{bmatrix}
\]

\[
\overline{E}_{i_{sca}}(r) = \overline{E}_{i}(r) \overline{\chi}(r)
\]

The scattered field then interacts with the Rx aperture

\[
g_{i,j}(\omega) = \int_{r} \overline{E}_{i}(r) \overline{\chi}(r) \, E_{j}^T(r) \, dr
\]

Polarimetric computational imaging

Forward model of the polarimetric imaging problem

\[ g_{i,j}(\omega) = \int_{\mathcal{R}} \overline{E_i(r)} \overline{\chi(r)} \overline{E_j(r)}^T \, dr \]

Operating with two panels

Operating with a single two-ports panel

Polarimetric computational imaging

Forward model of the polarimetric imaging problem

\[ g_{i,j}(\omega) = \int_{r} \overline{E}(r) \overline{\chi}(r) \overline{E}_j(r)^T \, dr \]

Understanding the physics behind the susceptibility tensors:
The electrons within the target medium are represented by a simple mechanical model.

\[ \overline{\chi}(r) = \overline{R}(r) \text{diag}(\overline{\xi}(r)) \overline{R}(r)^T \]

- Rotation matrix
- Diagonalized tensor
Polarimetric computational imaging

Eigendecompositions help to identify the directions of the main axis of the tensors (eigenvectors) and the associated eigenvalues define the anisotropic behavior of each voxel.

Isotropic scattering

Anisotropic scattering

Understanding the physics behind the susceptibility tensors:
The electrons within the target medium are represented by a simple mechanical model.

\[
\overline{\chi}(r) = \overline{R}(r) \text{diag}(\overline{\xi}(r)) \overline{R}(r)^T
\]

\[
\overline{R}(r) \quad \text{Rotation matrix}
\]

\[
\overline{\xi}(r) \quad \text{Diagonalized tensor}
\]
Polarimetric computational imaging

Analogy with diffusion tensor imaging

Magnetic Resonance Imaging

The main axis analysis of the tensors reconstructed in each voxel makes it possible to reconstruct the path of the fibers in the brain

Understanding the physics behind the susceptibility tensors:

The electrons within the target medium are represented by a simple mechanical model

\[ \overline{\chi}(r) = \overline{R}(r) \text{diag}(\overline{\xi}(r)) \overline{R}(r)^T \]

\( \overline{R}(r) \) Rotation matrix

\( \overline{\xi}(r) \) Diagonalized tensor


Polarimetric computational imaging

Forward model of the polarimetric imaging problem

\[ g_{i,j}(\omega) = \int \bar{E}_i(r) \bar{\chi}(r) \bar{E}_j(r)^T \, dr \]

Solving the inverse problem

The Tx and Rx vector fields are inversed to reveal the interaction between each couple of polarization states

\[ \hat{\chi}(r) = \bar{E}_i(r)^+ g_{i,j} \bar{E}_j(r)^+ \]

Criteria to be met to ensure successful reconstruction

The radiated fields in transmission and reception must be simultaneously orthogonal in three domains:

- in frequency
- in polarization
- spatially

Polarimetric computational imaging

Computing the spatial correlation for each pair of ports and polarizations

Polarimetric computational imaging

Experimental validation
Imaging targets made with thin copper wires

Number of frequency samples: 3601
Frequency range: 17.5-26.5 GHz

Fig. 10. Copper wire letters used as targets. Each letter is 15 x 9 cm².
Polarimetric computational imaging

Results with the letter D
Reconstructed from the measurement of a single frequency-domain signal

For each voxel
- 9 tensor elements (only 6 are independent)
- each tensor element is a complex value
The diversity of representations of such tensors remains unexplored for these applications

Special feature of the short-range polarimetric imaging
We obtain projections on the propagation axis

x and z are the transverse axis, y is the longitudinal (propagation) axis

Polarimetric computational imaging

Results with the letter D
Reconstructed from the measurement of a single frequency-domain signal

Extraction of a cross-section plan
The phase is included in the reconstructions and can be color-coded to reveal more information

The results are still based Born first approximation we must keep a critical eye on the analysis of the results

x and z are the transverse axis, y is the longitudinal (propagation) axis

Polarimetric computational imaging

Improvements with basic processing
Correlation of transverse components

\[ \hat{\chi}_{corr}(r) = \hat{\chi}_{xy}(r) \cdot \hat{\chi}_{yz}(r)^* \]

Optimized SNR
Anisotropic behavior

noise reduced by the coherent sum of the useful parts of the tensor
orientation of copper wires revealed by the phase information
Polarimetric computational imaging

Going further with eigendecompositions

The diagonalization of the tensors is computed for each voxel on the real and imaginary parts separately.

Polarimetric computational imaging

Going further with eigendecompositions
RGB color-coding according to the orientation of the main axis of each ellipsoid

Polarimetric phaseless imaging

In conclusion of this work

Conventional MIMO polarimetric system

Double the hardware constraints compared to systems based on the scalar approximation

Very few applications in short range imaging due to these limitations

Computational MIMO polarimetric system

Requires a single measurement chain provided that a full scan of the near fields is carried out

Paves the way for volumetric reconstruction of susceptibility tensors

Phaseless computational imaging

Conventional MIMO system

Phase measurement also creates heavy hardware constraints on each measurement chain

Motivation for this work

is it possible to reconstruct images from the intensity measurement of signals that interacted with a target?

In other words, can we replace our VNAs with spectrum analyzers and a little mathematics?

Are computational systems compatible with/supportive of these objectives?
Phaseless computational imaging

**First concern:** phase information is crucial

![Phaseless imaging example](image)

2D Fourier transform

Phase swap
Phaseless computational imaging

First concern: phase information is crucial

After the phase swap and an inverse 2D Fourier transform
**Phaseless computational imaging**

**Key element:** phase information can however be encoded in the intensity information

Signals received in an aperture in the presence of two isotropic point sources

![Diagram of two isotropic point sources](image)

Interferences of complex contributions:
Transformation of phase information into intensity fluctuation

**Inverse problem solving:** Phase retrieval algorithms
Phaseless computational imaging

**Objective:** Application to microwave imaging

Expression of the measured signals

\[ s(r_t, r_r) = \int G(r_t, r_r)\sigma(r)G(r, r_r) d^3r \]

Matrix formalism

\[ s = H\sigma \]

Reconstruction by pseudo-inversion

\[ \hat{\sigma} = H^+s \]

Intensity measurement: Reflectivity reconstruction?

\[ s_I = |s|^2 = |H\sigma|^2 \]
Phaseless computational imaging

Intensity measurement: Reflectivity reconstruction?

\[ s_I = |s|^2 = |H\sigma|^2 \]

Solving an optimization problem

\[
\begin{align*}
\min_{\sigma} & \quad f(\sigma) = \frac{1}{m} \sum_{i=1}^{m} |\langle h_i, \sigma \rangle|^2 - |s_i|^2 \\
& \quad = \| (H\sigma)^2 - |s|^2 \|_1
\end{align*}
\]

Convex programming
Example: CVX and Matlab

```matlab
m = 20; n = 10; p = 4;
A = randn(m,n); b = randn(m,1);
C = randn(p,n); d = randn(p,1);
e = rand;
cvx_begin
    variable x(n)
    minimize( norm( A * x - b, 2 ) )
    subject to
    C * x == d
    norm( x, Inf ) <= e
end
```

Iterative algorithm
Example: Truncated Wirtinger Flow

\[ y = |Ax|^2 \]

\[
x^{(t+1)} = x^{(t)} + \frac{\mu_t}{m} \sum_{i \in S_{t+1}} \nabla l_i(x^{(t)})
\]

\[
\nabla l_i(x^{(t)}) = 2 \frac{y_i - |a_i^* x^{(t)}|^2}{x^{(t)} \ast a_i}
\]

where the adaptive index set \( S_{t+1} \) is determined by Chen and Candès satisfying for any \( i \in S_t \):

\[
\frac{a_i^* x^{(t)}}{|a_i^* x^{(t)}|} \propto |x^{(t)}|\
\frac{y_i - |a_i^* x^{(t)}|^2}{a_i^* x^{(t)}} \leq \frac{1}{m} \frac{|y_i - |a_i^* x^{(t)}|^2|}{\|x^{(t)}\|}
\]
Phaseless computational imaging

Experimental validation: SAR PhaseLess imaging

Aperture size: 60 × 60 cm²
Number of samples: 61 × 61 = 3721
Background measurement (without marbles)

The results converge towards those obtained with phase measurement

Once again, the hardware constraint is transferred to the software layer
Phaseless computational imaging

Cavity-based phaseless localization
First step: localizing a source from an intensity measurement

The diversity of the magnitudes of near-field scans is the basis for encoding information

$\nu_1 = 23 \text{ GHz} - \nu_2 = 23.002 \text{ GHz}$
Phaseless computational imaging

Cavity-based phaseless localization
First step: localizing a source from an intensity measurement

Reconstruction from the measurement of two signals on the output ports of the cavity

The diversity of the magnitudes of near-field scans is the basis for encoding information

Algorithm: Truncated Wirtinger Flow
Number of spatial samples: $20 \times 20 \times 10$
Number of frequency samples: 3601
Frequency range: 17.5-26.5 GHz
Phaseless computational imaging

Cavity-based phaseless imaging

Switching to the interrogation of a reflectivity function

Small object for the determination of the point spread function

Use of a more advanced technique based on the sparsity of the scene to be imaged

Phaseless computational imaging

Cavity-based phaseless imaging

Switching to the interrogation of a reflectivity function

Key parameter: ratio $M/N$

- $M/N = 0.3$
- $M/N = 0.6$
- $M/N = 0.8$

Comparison of the performances achieved by the different approaches

Complex measurement
Sparse WF
Regular WF

Phaseless computational imaging

Complex-based reconstruction

Phaseless reconstruction

Phaseless computational imaging

Interesting feature of phaseless imaging:
relaxes the phase coherency requirements for applications where large apertures are needed

Without phase error

With phase error

Complex-Based

Without phase error

With phase error

Without phase error

With phase error

Without phase error

With phase error

STh-01

MODERN ADVANCES IN COMPUTATIONAL IMAGING AT MICROWAVE AND MILLIMETRE-WAVE FREQUENCIES

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