```
> restart : with(LinearAlgebra) : with(simplex) : with(PolynomialTools) : with(algcurves) :
  libname;
          "/Library/Frameworks/Maple.framework/Versions/18/lib", "."                (1)
> libname
     := "/Users/maddah/Documents/Work/Packages/packages/libraries/ParamInt/ParamInt.lib",
     "/Library/Frameworks/Maple.framework/Versions/18/lib";
libname :=                                                                            (2)
    "/Users/maddah/Documents/Work/Packages/packages/libraries/ParamInt/ParamInt.lib",
    "/Library/Frameworks/Maple.framework/Versions/18/lib"
> march('list',
     "/Users/maddah/Documents/Work/Packages/packages/libraries/ParamInt/ParamInt.lib");
[["TurnPtParam1.m", [2015, 9, 14, 1, 51, 39], 1024, 1021], ["BlockDiagNil.m", [2015, 9, 14,   (3)
    1, 51, 39], 2045, 555], ["MoserCriteriaParam.m", [2015, 9, 14, 1, 51, 39], 2600, 608],
    ["LeveltParamM.m", [2015, 9, 14, 1, 51, 39], 3208, 1429], ["SplitParam.m", [2015, 9, 14,
    1, 51, 39], 4637, 580], ["ExpOrderAndPolyParam.m", [2015, 9, 14, 1, 51, 39], 5217, 859],
    ["update_xi.m", [2015, 9, 14, 1, 51, 40], 6076, 299], ["NonzeroEigenvaluesParam.m",
    [2015, 9, 14, 1, 51, 40], 6375, 2350], ["ExpPartParam.m", [2015, 9, 14, 1, 51, 40], 8725,
    5796], ["TurnPtParam2.m", [2015, 9, 14, 1, 51, 39], 14521, 1069],
    ["AlgGaussCompleteBasis.m", [2015, 9, 14, 1, 51, 39], 15590, 303],
    ["SplitSysParam1.m", [2015, 9, 14, 1, 51, 39], 15893, 960], ["CompleteBasisParam.m",
    [2015, 9, 14, 1, 51, 39], 16853, 975], ["LeveltParam.m", [2015, 9, 14, 1, 51, 39], 17828,
    1249], ["ScalarPartParam.m", [2015, 9, 14, 1, 51, 40], 19077, 436],
    ["RefinedExpPolyParam.m", [2015, 9, 14, 1, 51, 40], 19513, 423], ["update_sigma.m",
    [2015, 9, 14, 1, 51, 40], 19936, 1284], ["NilpParam.m", [2015, 9, 14, 1, 51, 40], 21220,
    1234], ["ExpPartParam_Inner_Inter.m", [2015, 9, 14, 1, 51, 40], 45986, 3797],
    ["AlgMoser.m", [2015, 9, 14, 1, 51, 39], 26251, 1440], ["AlgMoserCriteria.m", [2015, 9,
    14, 1, 51, 39], 27691, 606], ["NewtonPolygon.m", [2015, 9, 14, 1, 51, 39], 28297, 1262],
    ["AlgMoserRowElim.m", [2015, 9, 14, 1, 51, 39], 29559, 1387], ["AlgCompleteBasis.m",
    [2015, 9, 14, 1, 51, 39], 30946, 795], ["GaussCompleteBasis.m", [2015, 9, 14, 1, 51, 39],
    31741, 295], ["SplitSysParam2.m", [2015, 9, 14, 1, 51, 39], 32036, 2800],
    ["MySylvesterEF.m", [2015, 9, 14, 1, 51, 39], 34836, 579], ["SeparateBlockParam1.m",
    [2015, 9, 14, 1, 51, 39], 35415, 1141], ["MoserParam.m", [2015, 9, 14, 1, 51, 39], 36556,
    2099], ["GaussColElimRational.m", [2015, 9, 14, 1, 51, 39], 38655, 1340],
    ["MoserRowElimParam.m", [2015, 9, 14, 1, 51, 39], 39995, 1445],
    ["MoserRowElimParamh1.m", [2015, 9, 14, 1, 51, 39], 41440, 1944],
    ["NewtonRootsParam.m", [2015, 9, 14, 1, 51, 40], 43384, 1295], ["correct_sigma.m",
    [2015, 9, 14, 1, 51, 40], 44679, 759], ["test_stretch.m", [2015, 9, 14, 1, 51, 40], 45438,
    548]]
>
```

>

Weber'S equation eps^2 y"=x^2 y

>

>

> $A := \frac{1}{\text{epsilon}} \cdot Matrix\left(2, 2, \left[0, 1, x^2, 0\right]\right); ExpPartParam(A, x, t, \text{epsilon}, s, \text{omega}, t\_omega, 7);$

$$A := \begin{bmatrix} 0 & \frac{1}{\varepsilon} \\ \frac{x^2}{\varepsilon} & 0 \end{bmatrix}$$

"Line 446: The system ", $\begin{bmatrix} 0 & \frac{1}{\varepsilon} \\ \frac{x^2}{\varepsilon} & 0 \end{bmatrix}$, " is under reduction with dimension =", 2, " and ", ε, "-rank = ", 1, ". The variable is renamed as ", x, " and the parameter is renamed as ", ε

"Line 455: The system went through ", ε, "-rank reduction. The resulting system is of ", ε, "-rank ", 1, " and is given by ", $\begin{bmatrix} 0 & \frac{1}{\varepsilon} \\ \frac{x^2}{\varepsilon} & 0 \end{bmatrix}$

"Line 460: The system went through nonzero constant eigenvalue testing: resolution of turning point (if it exists), adjusting of sigma, and decoupling (Splitting Lemma). The resulting info (part to be processd for Exp part, remaining decoupled systems, new ranks) is given by   ",

$$\left[\left[\left[\frac{1}{t\,\varepsilon}, 1\right], \left[-\frac{1}{t\,\varepsilon}, 1\right], [0, 1]\right], \left[\left[1, \left[-\frac{1}{128}\frac{\varepsilon^3}{x} + \frac{1}{8}\frac{\varepsilon}{x} - \frac{1}{2\,x}\right], t, -2, 1\right], \left[1, \left[\frac{1}{128}\frac{\varepsilon^3}{x} - \frac{1}{8}\frac{\varepsilon}{x} - \frac{1}{2\,x}\right], t, -2, 1\right], \left[0, [\;\;], t, -2, 1\right]\right], [0, 0, -\infty]\right]$$

"Line 446: The system ", $\left[-\frac{1}{128}\frac{\varepsilon^3}{x} + \frac{1}{8}\frac{\varepsilon}{x} - \frac{1}{2\,x}\right]$, " is under reduction with dimension =", 1, " and ", ε, "-rank = ", 0, ". The variable is renamed as ", x, " and the parameter is renamed as ", ε

"Line 451: It is of dimension 1, and so the part ", 0, " is isolated to go through the exponential part and the remaining part ", $-\frac{1}{128}\frac{\varepsilon^3}{x} + \frac{1}{8}\frac{\varepsilon}{x}$

$-\frac{1}{2\,x}$, "is returned"

"Line 446: The system ", $\left[\ \frac{1}{128}\,\frac{\varepsilon^3}{x}-\frac{1}{8}\,\frac{\varepsilon}{x}-\frac{1}{2\,x}\ \right]$,

" is under reduction with dimension =", 1, " and ", ε, "-rank = ", 0,

". The variable is renamed as ", x, " and the parameter is renamed as ", ε

"Line 451: It is of dimension 1, and so the part ", 0,

" is isolated to go through the exponential part and the remaining part ", $\frac{1}{128}\,\frac{\varepsilon^3}{x}-\frac{1}{8}\,\frac{\varepsilon}{x}$

$-\frac{1}{2\,x}$, "is returned"

"Line 446: The system ", $[\ \ ]$, " is under reduction with dimension =", 0, " and ", ε, "-rank = ",

$-\infty$, ". The variable is renamed as ", x, " and the parameter is renamed as ", ε

"REDUCTION OF THE OUTER SYSTEM HAS FINISHED!"

" RESULT:"

"EXPONENTIAL PART OF OUTER SYSTEM"

"The following list contains the Exponential part in ", ε,

" with its relevant information: The ramification ", t, " in ", x, " and ramification ", s, " in ", ε,

", an entry in the exponential part (after the substitution of sigma and integration w.r.t. ", t,

"), sigma, and the multiplicity of this entry in the exponential part. We remark that the list might contain repititive mentioning of conjugates."

$$\left[\left[t,\,s,\,\frac{1}{2}\,\frac{t^2}{s},\,-2,\,1\right],\left[t,\,s,\,-\frac{1}{2}\,\frac{t^2}{s},\,-2,\,1\right]\right]$$

"SUBSYSTEMS"

"The following list contains the regularly-perturbed or unperturbed subsystems resulting from reduction, with their relevant information, following the order in the first list: The ramification

", t, " in ", x, " and ramification ", s, " in ", ε, ", sigma, Poincare rank in ", t,

" and the remaining matrix (sigma not substituted)"

$$\left[\left[t,\,s,\,-2,\,1,\,-\frac{1}{128}\,\frac{s^3}{t}+\frac{1}{8}\,\frac{s}{t}-\frac{1}{2\,t}\right],\left[t,\,s,\,-2,\,1,\,\frac{1}{128}\,\frac{s^3}{t}-\frac{1}{8}\,\frac{s}{t}-\frac{1}{2\,t}\right]\right]$$

"INTERMEDIATE AND INNER SYSTEMS"

"We have anticipated from the above reduction the following slopes in the (", x, ε,
")-Polygon (Iwano-Sibuya Polygon of the equivalent scalar equation): "

$$\{-2\}$$

"We now proceed by applying the streching transformation(s)", $\left\{\frac{1}{2}\right\}$,
"to the initial input system to obtain solutions in inner (and possibly intermediate) domain(s)."

"Upon applying the stretching ", x, " = ", $\sqrt{\varepsilon}\,\omega$, " and applying the ramification ", ε, " = ", s^2,
" , and renaming ", s, " to ", ε, " we get the following system:"

$$\varepsilon\,\omega, \begin{bmatrix} 0 & \frac{1}{\varepsilon} \\ \varepsilon\,\omega^2 & 0 \end{bmatrix}$$

" We apply the reduction algorithm which gives:"

" RESULT:"

"EXPONENTIAL PART OF INNER (OR INTERMEDIATE) SYSTEM"

"The following list contains the Exponential part in ", ε,
" with its relevant information: The ramification ", *t_omega*, " in ", ω, " and ramification ", s,
" in ", ε,
", an entry in the exponential part (after the substitution of sigma and integration w.r.t. ",
t_omega,
"), sigma, and the multiplicity of this entry in the exponential part. We remark that the list might contain repititive mentioning of conjugates."

$$[[t\_omega, s^2, 0, 0, 2]]$$

"SUBSYSTEMS"

"The following list contains the regularly-perturbed or unperturbed subsystems resulting from reduction, with their relevant information, following the order in the first list: The ramification

", *t_omega*, " in ", ω, " and ramification ", *s*, " in ", ε, ", sigma, Poincare rank in ", *t_omega*, " and the remaining matrix (sigma not substituted)"

$$\left[\left[t\_omega, s^2, 0, 0, \begin{bmatrix} 0 & t\_omega^2 \\ 1 & 0 \end{bmatrix}\right]\right]$$

"We have anticipated from the above reduction the following additional slopes in the (", ω, ε, ")-Polygon (Iwano-Sibuya Polygon of the equivalent scalar equation) for the initial input system: "

$$\{\ \}$$

"FINAL SET OF SLOPES FOR THE INITILA INPUT SYSTEM"

$$\{-2\} \tag{4}$$

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