CHSH INEQUALITY FOR A SINGLE QUTRIT

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TOOLS

See [1] and [5]
Qubit (element of \( \mathbb{C}^2 \)) is implemented by spin \( \frac{1}{2} \) photon.

- Biphoton’s space is the three dimensional space orthogonal to Singlet (generated by the three other Bell states, invariant when left and right qubits are permuted): \( \text{Sym}(\mathbb{C}^2 \otimes \mathbb{C}^2) \cong \mathbb{C}^3 \)

Moreover we can make this correspondence:

\[
\begin{align*}
|00\rangle & \mapsto |+1\rangle; \\
|1\rangle \mapsto |0\rangle; \\
|11\rangle & \mapsto |1\rangle.
\end{align*}
\]

Purpose:
- Show/Study how CHSH-2 can be pushed to a Single Qutrit by symmetrisation
- New view of non-classical behavior of Qutrit

CARRIER = SYMMETRISATION [4]

\[
U(\mathbb{C}^2) \quad \rightarrow \quad \Gamma(V) = \frac{1}{2}(V \otimes I + I \otimes V); \text{ with eigenvalue in \{−1, 0, 1\}}
\]

CHSH-2; \( d=2 \):

Bell expression

- One partie, \( n=1 \) : \( \mathbb{C}^2 \), using non-commuting observables, with any state:

\[
\langle \Psi| A_1 B_1 + A_1 B_2 + A_2 B_1 - A_2 B_2 |\Psi\rangle
\]

- Two parties, \( n=2 : \mathbb{C}^2 \otimes \mathbb{C}^2 \):

\[
\langle \Psi| A_1 \otimes B_1 + A_1 \otimes B_2 + A_2 \otimes B_1 - A_2 \otimes B_2 |\Psi\rangle
\]

\[
|\Psi\rangle = \sqrt{2}(|00\rangle + |11\rangle)
\]

Setting for violation

\[
\begin{align*}
A_1 &= Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\
A_2 &= X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\
B_1 &= \frac{Z \pm X}{\sqrt{2}} = H \\
B_2 &= \frac{Z - X}{\sqrt{2}} = K
\end{align*}
\]

CHSH-2; \( d=3 : \mathbb{C}^3 \):

We have:

\[
\Gamma(A_1)\Gamma(B_1) + \Gamma(A_1)\Gamma(B_2) + \Gamma(A_2)\Gamma(B_1) - \Gamma(A_2)\Gamma(B_2)
\]

With the same choice of observable as in \( d = 2 \), biphoton \( \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \) or equivalently qutrit \( \frac{1}{\sqrt{2}}(|1\rangle + |1\rangle + |−1\rangle) \) gives expectation value \( 2\sqrt{2} \) that implies violation of \( \sqrt{2} \)

Compared to KCBS inequality on one Qutrit which gives violation of \( 5 - 2\sqrt{5} \), we have better violation.

We use non commutative operators.

REFERENCES


