CHSH inequality for a single qutrit

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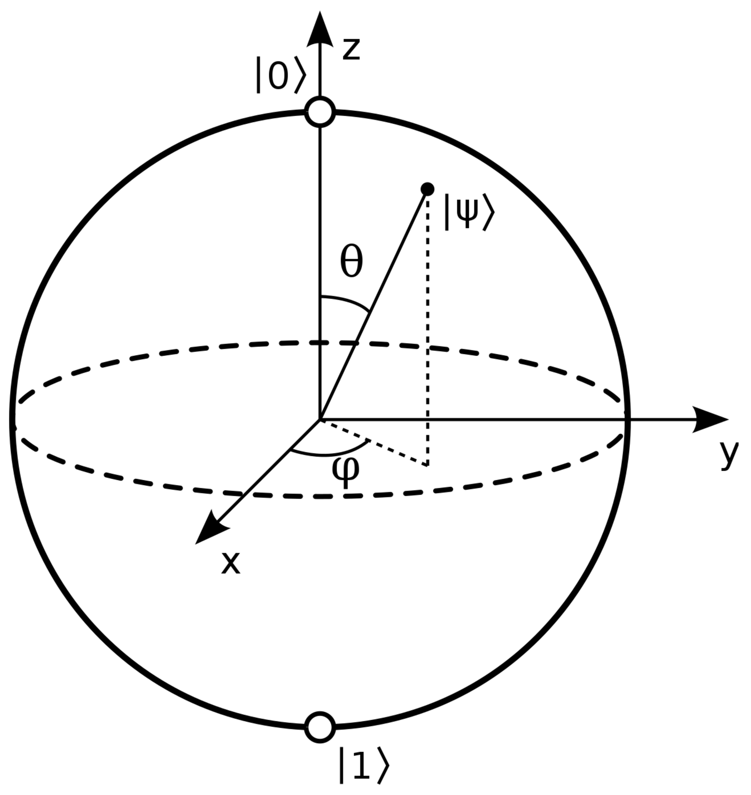
Tools

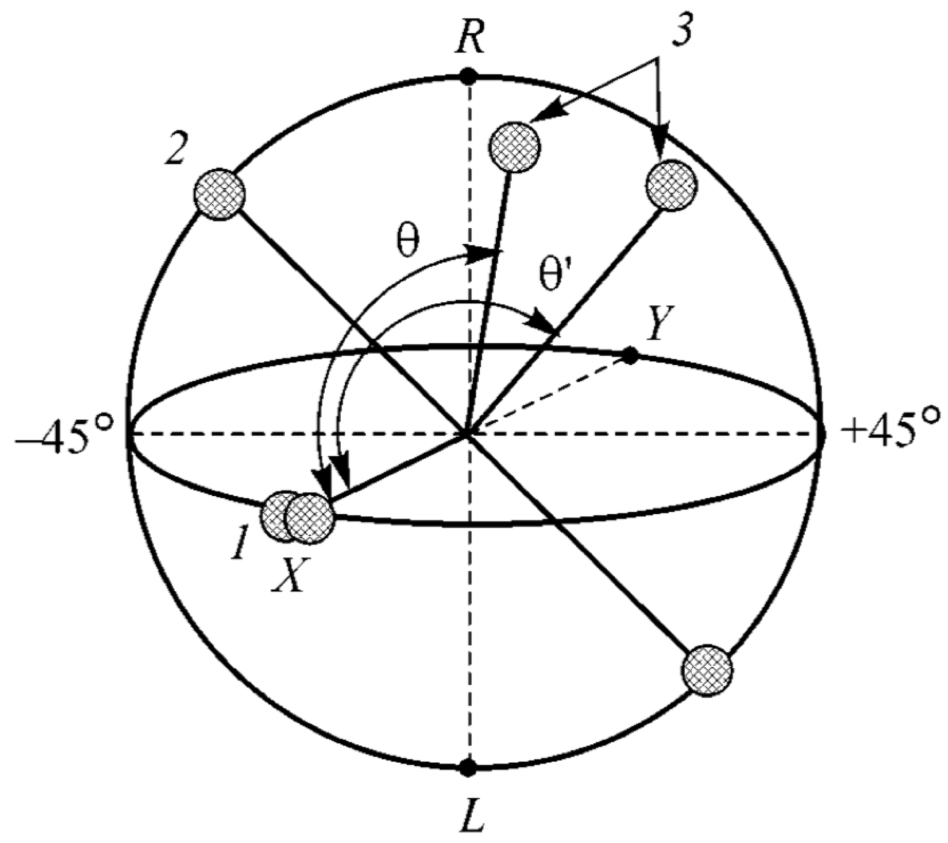
See [1] and [5]

Qubit (element of $\mathbb{C}^2$) is implemented by spin $\frac{1}{2}$ photon.



Bloch' Sphere

Qutrits (element of $\mathbb{C}^3$) are often implemented by biphotons . (intricated and indistinguishable photon pair)



Poincaré' Sphere

- Biphoton'space is the three dimensional space orthognal to Singlet (generated by the three other Bell states, invariant when left and right qubits are permuted): $Sym(\mathbb{C}^2 \otimes \mathbb{C}^2) \simeq \mathbb{C}^3$

Moreover we can make this correspondence : $|00\rangle \mapsto |+1\rangle$; $(1/\sqrt{2})(|10\rangle + |01\rangle) \mapsto |0\rangle$; $|11\rangle \mapsto |-1\rangle$

Purpose :

- Show/Study how CHSH-2 can be pushed to a **Single Qutrit** by symmetrisation
- New view of non-classical behavior of Qutrit

Carrier = Symmetrisation [4]

$$U(\mathbb{C}^2) \to \text{operators over } Sym(\mathbb{C}^2 \otimes \mathbb{C}^2) \simeq \mathbb{C}^3$$

$$\Gamma : \quad V \mapsto \Gamma(V) = \tfrac{1}{2}(V \otimes I + I \otimes V)$$; with eigenvalue in $\{-1, 0, 1\}$

CHSH-2 ; d=2

Bell expression

- One partie, n=1 : $\mathbb{C}^2$, using non-commuting observables, with any state :

$$\langle \Psi | A_1 B_1 + A_1 B_2 + A_2 B_1 - A_2 B_2 | \Psi \rangle$$

- Two parties, n= 2 : $\mathbb{C}^2 \otimes \mathbb{C}^2$:

$$\langle \Psi | A_1 \otimes B_1 + A_1 \otimes B_2 + A_2 \otimes B_1 - A_2 \otimes B_2 | \Psi \rangle$$

$|\Psi\rangle = 1/\sqrt{2}(|00\rangle + |11\rangle)$

Setting for violation

$$A_1 = Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad A_2 = X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad B_1 = \frac{Z+X}{\sqrt{2}} = H$$

$$B_2 = \frac{Z-X}{\sqrt{2}} = K$$

CHSH-2 ; d=3 : $\mathbb{C}^3$

We have

$$\Gamma(A_1)\Gamma(B_1) + \Gamma(A_1)\Gamma(B_2) + \Gamma(A_2)\Gamma(B_1) - \Gamma(A_2)\Gamma(B_2)$$

With the same choice of observable as in $d = 2$, biphoton $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ or equivalently qutrit $\frac{1}{\sqrt{2}}(|+1\rangle + |-1\rangle)$ gives expectation value $2\sqrt{2}$ that implies violation of $\sqrt{2}$

Compared to KCBS inequality on one Qutrit which gives violation of $5 - 2\sqrt{5}$, we have better violation.
We use non commutative operators.

References

[1] A.V. Burlakov, M.V. Chekhova, O.A. Karabutova, D.N. Klyshko, S.P. Kulik: "Polarization state of biphotons: quantum ternary logic." Phy Review A 60, R4209 (1999)

[2] D. Kaszlikowski, L.C; Kwek, J.L Chen, M.Zukowski, C.H. Oh : *Clauser-Horne inequality for three-state systems*. Physical Review A 65, 032118 (2002).

[3] A.A. Klyachko, M.A. Can, S. Binicioglu, A.S. Shumovsky: : *simple test for hidden variables in spin-1 systems* . Physical Review Letters 101, 020403 (2008).

[4] P. Kurzynski, A. Kolodziejski, W. Laskowski, M. Markiewicz: *Three-dimensional visualization of a qutrit*. Physical Review A 93, 062126 (2016).

[5] B.P. Lanyon, T.J. Weinhold, N.K. Langford, J.L. O'Brien, K.J. Resch, A. Gilchrist, A.G. White: *Manipulating biphotonic qutrits.*. Phy Review Letters 100, 060504 (2008).