Heterogeneous time preferences

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Philippe Darreau
CRMM,University of Limoges darraeu@unilim.fr

François Pigalle
LEA, University of Limoges pigalle@unilim.fr

Abstract

An embarrassing requirement of the Solow-Ramsey growth model is that in steady state all the agents must have the same time preference. It is shown that individuals having a weak potentiality for physical capital accumulation (i.e. too impatient), can have the same growth rate as other individuals, provided they have a high potentiality of human capital accumulation.

Keywords : growth; time preference; optimal saving rate; representative agent

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1. Introduction

An embarrassing requirement of the Solow-Ramsey growth model is that in steady state all the agents must have the same time preference. This condition is frustrating as differences in rates of time preference across individuals can't be taken into consideration. This implies that the optimal saving rate used by economic theory is not a concept based on methodological individualism. In fact individuals don't show the same patience, and consideration for various given psychologies is a main feature of the neoclassical approach.

Becker (1980) showed that heterogeneity is impossible, as was suggested by Cantillon (1755, la parabole des soldats) and by Ramsey (1928). Over an infinite horizon, the highest saver who has the lowest time preference will hold all the capital. The others will obtain only their wage income. Our purpose is to show that within a model over an infinite planning horizon for each dynasty, it is possible to achieve an aggregation consistent with heterogeneous time preferences.

Of course if the only difference between dynasties consists in their time preferences, it is impossible to find a steady state, that is a state where consumption growth rates of all dynasties are equal. In this case, parsimonious dynasties would have higher growth rates than extravagant dynasties. A low rate of time preference gives a natural advantage to accumulating physical capital and results in a higher consumption growth rate. Dynasties (countries) must of course differ in at least two compensating features in order to produce the same growth rates.

In an endogenous human capital driven growth model, it seems reasonable to us to admit a human capital accumulation specific to each dynasty , the « human capital accumulation potentialities » being a dynastic feature. In our economy with single good and homogeneous human capital, we have dynasties of carpenters, bankers, musicians and managers. We suppose that the human capital accumulation potentialities, specific to each trade, are different, and that the productivity parameter of the human capital production function (q_i) is specific to each dynasty.

Time preference can be assimilated to physical capital accumulation potentiality. Our purpose is to show that a dynasty (an economy) having both a high physical capital accumulation potentiality and a low human capital accumulation potentiality, can have the same consumption growth rate as a dynasty (an economy) having a low rate of time preference and a high educational productivity. We show that there is a steady state resulting from the aggregation of agents' behavior. These agents differ in relation to their respective human and physical capital accumulation potentiality. In this paper we propose first a simple model enabling heterogeneity. Then a parametrized simulation shows that aggregation is possible. The optimal saving rate of economy is then correctly defined as the aggregation of individual saving rates.

2. The model

Rebelo (1991) and Mulligan & Sala-i-Martin (1992) have tabled an endogenous growth model including human capital accumulation requiring two inputs: human capital and physical capital. We adapt this model in order to deal with the problem of heterogeneity.

Assumption 1 : Each agent has a pure rate of time preference, which is peculiar to each of them but they all have the same intertemporal elasticity of substitution. The intertemporal utility function of each agent (i) is:

$$U_i = \int_{t=0}^{+\infty} e^{-r_i t} \frac{c_i^{1-s} - 1}{1-s} dt \tag{1}$$

Assumption 2 : Each of the n agents owns and accumulates physical capital as well as human capital.

Each agent (i) owns a physical capital k_i a part of which v_i is used to accumulate human capital, the remainder $(1-v_i)$ being used to produce goods. In the same way, each agent owns human capital h_i a part of which u_i is used to accumulate human capital, the remainder $(1-u_i)$ being allocated to work.

This assumption enables the introduction of two additional control variables v_i and u_i. Should the agent accumulate only physical capital as in the Solow-Ramsey model, his unique control variable would be c_i ; arbitration would be carried out according to his rate of time preference. Both accumulation rate and consumption growth rate would be the same as those of others agents only where his rate of time preference was equal to that of other agents. If the agents can accumulate human capital, they have a second control variable (u_i and v_i are linked) giving the model an additional degree of freedom.

The macroeconomic production function is:

$$Y = A\left(\sum_i (1-v_i)k_i\right)^b \left(\sum_i (1-u_i)h_i\right)^{1-b} \tag{2}$$

Each agent has an income resulting from the macroeconomic production function of the good which remunerates his physical capital and his work. This remuneration is at marginal productivity.

$$r_k = bA\left(\frac{\sum_i (1-u_i)h_i}{\sum_i (1-v_i)k_i}\right)^{1-b} \tag{3}$$

$$r_h = (1-b)A\left(\frac{r_k}{bA}\right)^{\frac{b}{b-1}} \tag{4}$$

An agent's accumulation of physical capital equals his income less his consumption:

$$Dk_i = r_k(1-v_i)k_i + r_h(1-u_i)h_i - c_i \tag{5}$$

Assumption 3 : Each agent accumulates human capital by means of a dynastic accumulation function. In this function, the utilization of physical capital has decreasing point-in-time returns.

$$Dh_i = q_i\left(v_i^f k_i\right)^a \left(u_i^y h_i\right)^{1-a} \quad \text{with } \phi < 1 \text{ et } \psi \leq 1 \tag{6}$$

where **q_i** is the human capital accumulation potentiality of the dynasty (i). The productivity parameter of the human capital production function (q_i) is specific to each family or each country. This can be considered as a psychological factor inherent in the educational ability of the family but also as a technological factor which depends on the performance of the country's educational system (Pigalle (1994)), as well as on the technical specificity of the training processes in various trades. Introducing decreasing point-in-time returns ($\phi < 1$) in the production function of human capital (Mulligan & Sala-i-Martin (1992)), makes the intertemporal process of investment in human capital sensitive to investment chronology.

To maximize (1) under constraints (5) (6), the agent controls v_i, u_i, c_i, marginal productivities of physical and human capital being given. The Hamiltonian is :

$$H(.) = e^{-r_i t} \frac{c_i(t)^{1-s} - 1}{1-s} + l_{k_i}\left(r_k(1-v_i)k_i + r_h(1-u_i)h_i - c_i\right) + q_{h_i} q_i \left(v_i^f k_i\right)^a \left(u_i^y h_i\right)^{1-a}$$

Conditions for optimization are:

$$\frac{¶H[.]}{¶c_i} = 0 \quad \Rightarrow \quad \frac{Dc_i}{c_i} = \frac{1}{s}\left(-\frac{D\,l_{k_i}}{l_{k_i}} - r_i \right) \qquad \text{(c-1)}$$

$$\frac{¶H[.]}{¶v_i} = 0 \quad \Rightarrow \quad -\,l_{k_i}\; r_k\; k_i + q_{h_i}\; q_i\; \frac{¶g[.]}{¶v_i} = 0 \qquad \text{(c-2)}$$

$$\frac{¶H[.]}{¶u_i} = 0 \quad \Rightarrow \quad -\,l_{k_i}\; r_h\; h_i + q_{h_i}\; q_i\; \frac{¶g[.]}{¶u_i} = 0 \qquad \text{(c-3)}$$

$$\frac{¶H[.]}{¶k_i} = -D\,l_{k_i} \quad \Rightarrow \quad -\,D\,l_{k_i} = l_{k_i}\; r_k (1 - v_i) + q_{h_i}\; q_i\; \frac{¶g[.]}{¶k_i} \qquad \text{(c-4)}$$

$$\frac{¶H[.]}{¶h_i} = -Dq_{h_i} \quad \Rightarrow \quad -\,Dq_{h_i} = l_{k_i}\; r_h (1 - u_i) + q_{h_i}\; q_i\; \frac{¶g[.]}{¶h_i} \qquad \text{(c-5)}$$

$$\lim_{t \to +\infty} l_i\; k_i = 0 \qquad \text{(c-6)}$$

$$\lim_{t \to +\infty} q_i\; h_i = 0$$

From (c-2)/(c-3), we can extract the implicit price ratio, then by replacing it in (c-4) we obtain :

$$-\frac{D\,l_i}{l_i} = r_k \left(1 + v_i\, \frac{1-f}{f} \right) \qquad (7)$$

When $\phi < 1$, the decreasing rate of the implicit price of physical capital becomes greater than the marginal productivity of physical capital. Therefore we obtain our result:

$$\frac{Dc_i}{c_i} = \frac{1}{s}\left(r_k \left(1 + v_i\, \frac{1-f}{f} \right) - r_i \right) \qquad (8)$$

The agent consumption growth rate depends on: marginal productivity (ρ_k) which must be unique (unique price law), psychological features (ρ_i, σ), and control variables specific to each individual (v_i). Thus, there are couples (ρ_i,v_i) which give each agent a similar growth rate in steady state. The value of the variable v_i is determined in steady state both by the value of macroeconomic parameters and that of individual parameters (q_i et ρ_i): $v_i = v_i (\alpha, \beta, \phi, \psi, \sigma, A, q_i, \rho_i)$.

It only remains to find couples (q_i et ρ_i) that ensure the same growth rate for each agent in steady state.

3. Parametrization and aggregation of the optimal saving rate in the economy

The steady state is characterized by : i) $g = \frac{Dc_i}{c_i} = \frac{Dk_i}{k_i} = \frac{Dh_i}{h_i} = \text{constant } \forall \text{ i}$

ii) $u_i, v_i, \frac{h_i}{k_i}, \frac{c_i}{k_i}, \frac{k_i}{k_j} = \text{ constant } \forall\, i$

To solve this system, it must be transformed to find new state and control variables which are constant in steady state.

We define : $z_i = h_i / k_i$, $m_i = c_i / k_i$ and $a_i = k_i / k_1$

Then, dividing (c-2) by (c-3) we find :

$$u_i = \frac{y\,(1-a)}{fa}\, \frac{r_k}{r_h}\, \frac{v_i}{z_i} \qquad (9)$$

The steady state is the solution of: $$\begin{cases} Dz_i = f_{z_i}(z_i, m_i, v_i, r_k) = 0 \\ Dm_i = f_{m_i}(z_i, m_i, v_i, r_k) = 0 \\ Dv_i = f_{v_i}(z_i, m_i, v_i, r_k) = 0 \\ Dr_k = f_{r_k}(z_a, z_b, v_a, v_b, r_k, a_i) = 0 \\ Da_i = f_a(z_a, m_a, v_a, z_b, m_b, v_b, a_i, r_k) = 0 \end{cases} \quad (10)$$

In order to simplify we suppose that there are two agents; Alice and Bob (indexed by a and b). To solve the system with 8 equations and 8 unknowns, we propose the following values for exogenous parameters:

β	α	ϕ	ψ	σ	A	ρ_a	ρ_b	q_a	q_b
0,3	0,25	0,8	0,8	2	0,235	0,06	0,07	0,171	0,203

Alice has the lowest rate of time preference and the lowest human capital accumulation potentiality. Alice has more facilities than Bob to accumulate physical capital but more difficulty to accumulate human capital.

The steady state values of marginal productivity in physical and human capital as well as values of the variables z, m, v and u, can be deduced [1]:

r_k = 0.1200	r_h = 0,1309

Variables	z*	m*	v*	u*
Alice	1,037	0,197	0,05	0,157
Bob	8,632	1,009	0,383	0,145

From the values of the variables, the growth rate of implicit prices and consumption, together with the saving rate of agents, can be easily calculated. The latter is determined by : $s_i = \dfrac{Dk_i / k_i}{r_k(1-v_i) + r_h(1-u_i)z_i}$

Variables	$D\lambda_k / \lambda_k$	$D\theta_h / \theta_h$	Dc / c	s
Alice	-12,15 %	-12,15 %	3,075 %	13,46 %
Bob	-13,15 %	-13,15 %	3,075 %	2,95 %

It is no surprise that Alice, who has lowest rate of time preference, also has the highest saving rate, the lowest (h/k) ratio, the lowest (c/k) ratio and the lowest v.

In order to aggregate the model, we need the level of variables k_i, h_i and c_i. Provided k_a is given, we can deduce k_b, h_a and h_b, as shown in the following equation : $r_k = bA\left(\dfrac{(1-u_a)z_a k_a + (1-u_b)z_b k_b}{(1-v_a)k_a + (1-v_b)k_b}\right)^{1-b}$

Therefore where k_a = 800 at a given time, the following values are obtained :

Values for steady state at the time t= $\hat{t}$;

	k($\hat{t}$)	h($\hat{t}$)	c($\hat{t}$)	y($\hat{t}$)	Dk_i($\hat{t}$)
Alice	800	829	158,11	182,71	24,60
Bob	152,85	1314,21	154,28	158,98	4,70

To aggregate the saving rate we use the following equations :

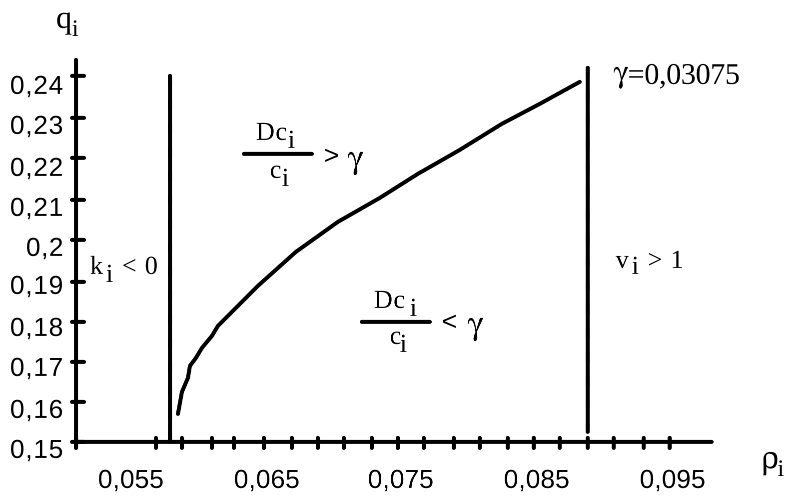
$$v_s = 1 - \frac{(1-v_a)k_a + (1-v_b)k_b}{k_a + k_b} = 0,1035 \qquad s_s = \frac{Dk_a + Dk_b}{y_a + y_b} = \frac{b\,g}{(1-v_s)r_k} = 8,574\%$$

The social optimal saving rate concept stemming from representative agent models is not justified in terms of methodological individualism. The optimal saving rate of our model is defined as the aggregation of the dynasties' optimal saving rates, and is more consistent with methodological individualism.

5. Conclusion

Individuals can have a low physical capital accumulation potentiality (high ρ_i) and have the same growth rate as the others, providing that they have a high human capital accumulation potentiality (high q_i). Equal growth consumption rate for heterogeneous agents can be satisfied provided ρ_i and q_i correspond to the following isogrowth curve:

couples (ρ_i, q_i) with isogrowth :



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[1] For calculations we take $r_k = 0{,}12$ then we calculate the respective q_i for each agent. In the text we present the logical process : With q_i given, we « find » r_k.