Lattice-based Cryptography

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Advisors: *P. Gaborit*[1] *& C. Aguilar Melchor*[1]

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123 avenue Albert Thomas
87060 Limoges CEDEX, France

XLIM - 4th Students Workshop
September the 25th, 2014

Outline

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Fundamental Goals

<u>Context:</u> 

wants to send message  to 

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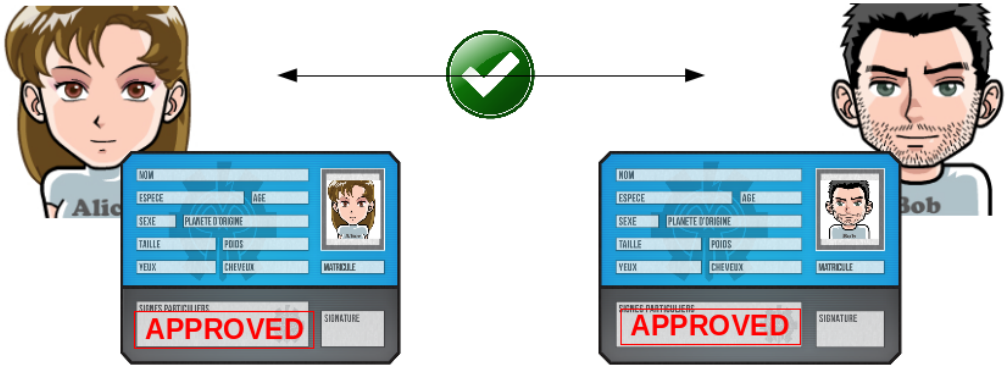
Cryptography aims for ensuring:

- Authentication
- Confidentiality
- Integrity
- Non-Repudiation



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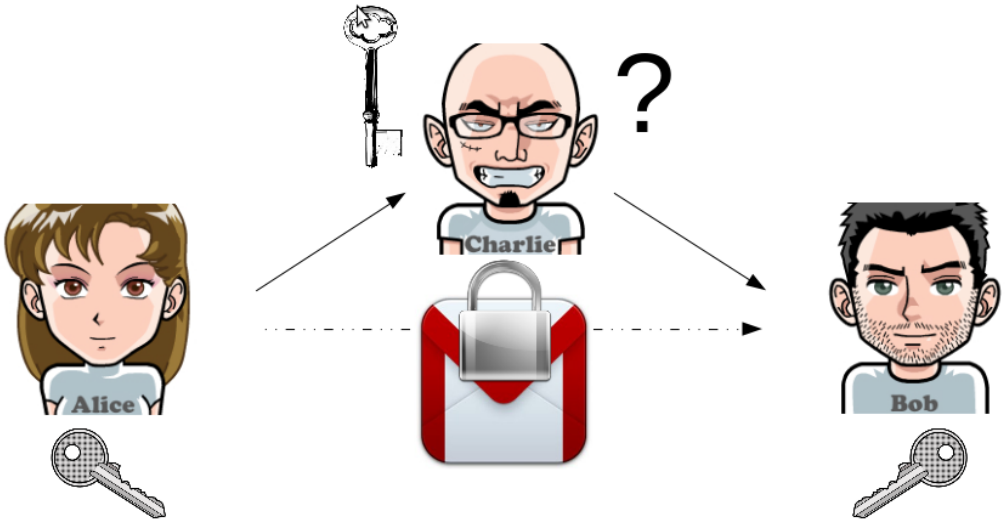

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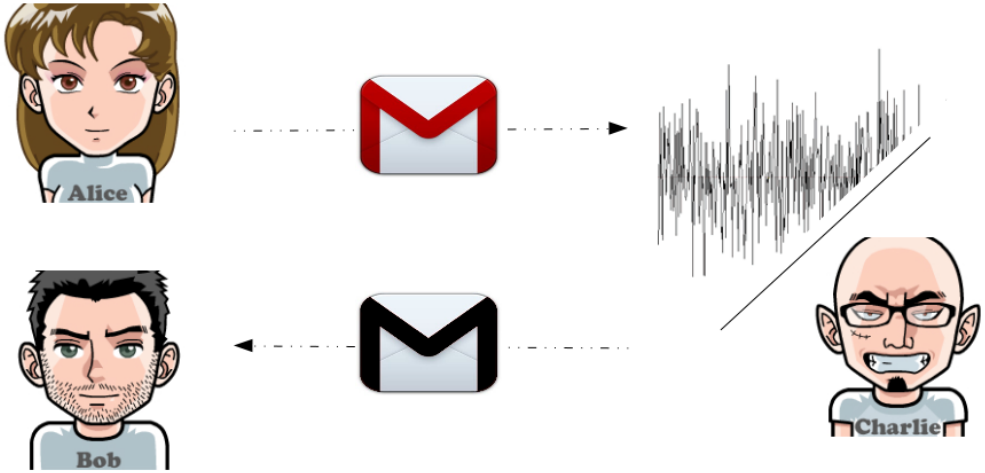

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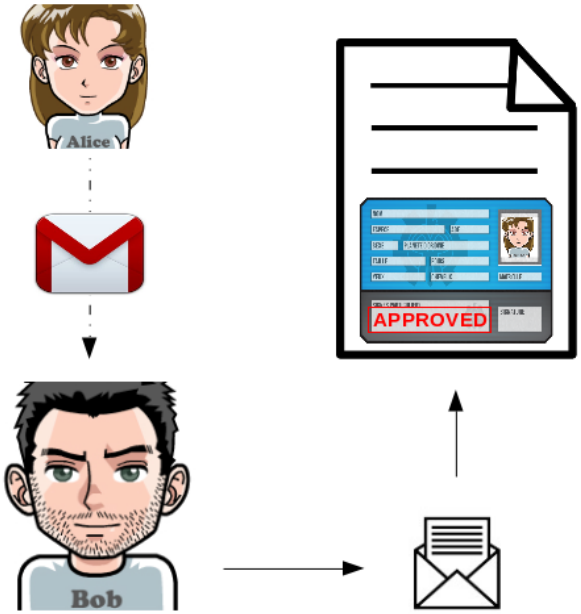

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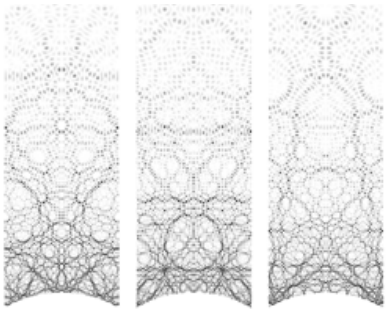
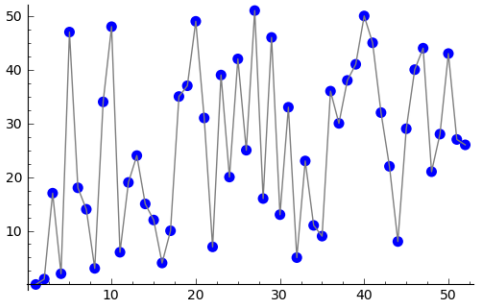
Techniques and Limitations

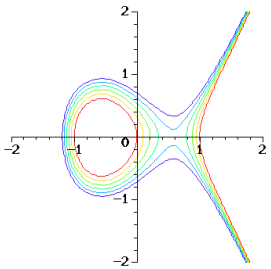
Big Integer Factorization



Finite Field Discrete Logarithm



Elliptic Curves Discrete Logarithm



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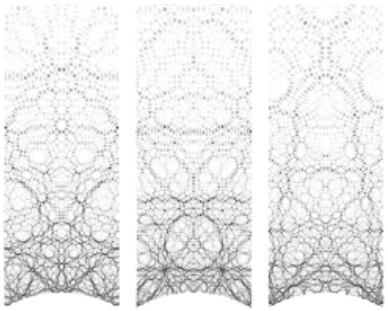

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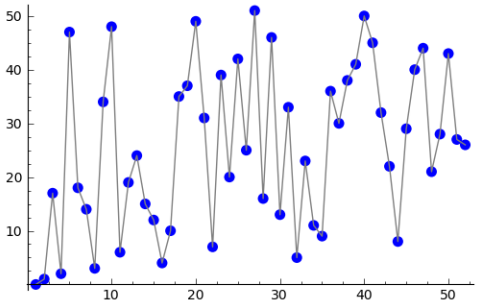

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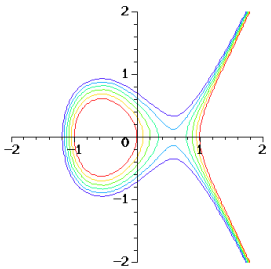

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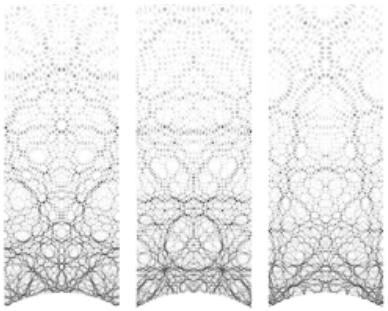

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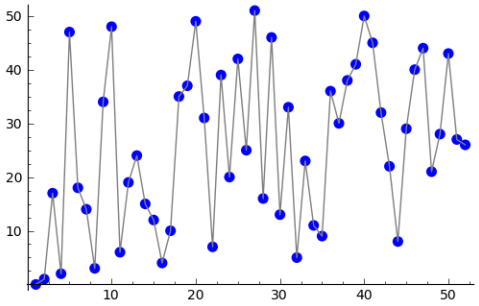

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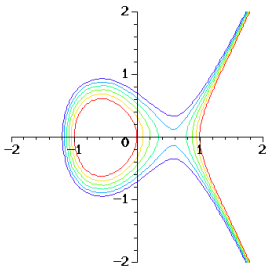

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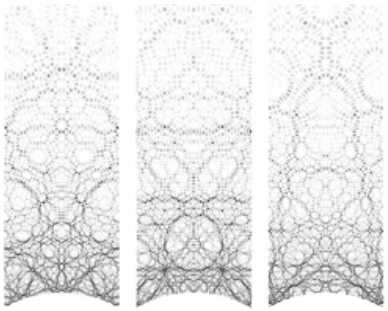

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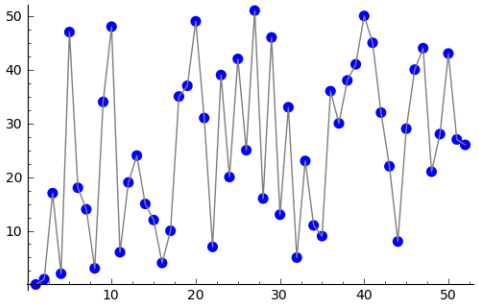

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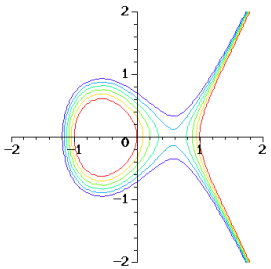

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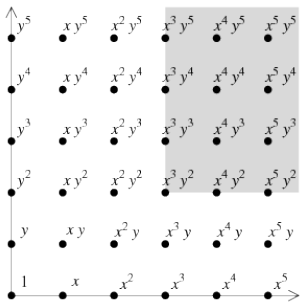
- ALREADY BROKEN BY QUANTUM COMPUTERS ! [Shor94]

Outline

Candidates

Multivariate



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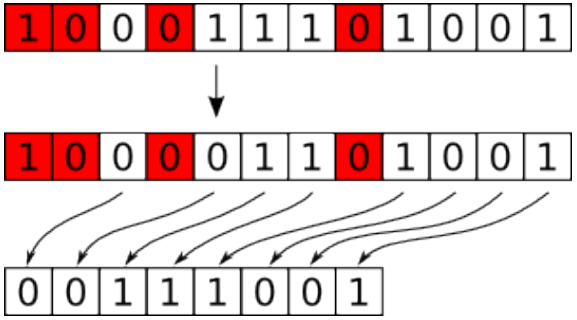
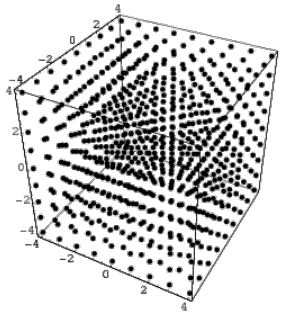


Euclidian Lattices



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Drawback

- Scarce: alternative crypto isn't implemented everywhere

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- Pretty new schemes: not (yet) efficient enough to be practical BUT many ways to improve them

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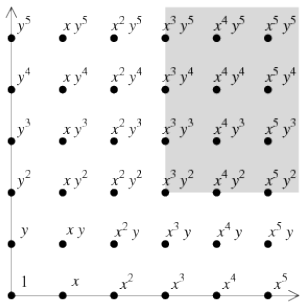

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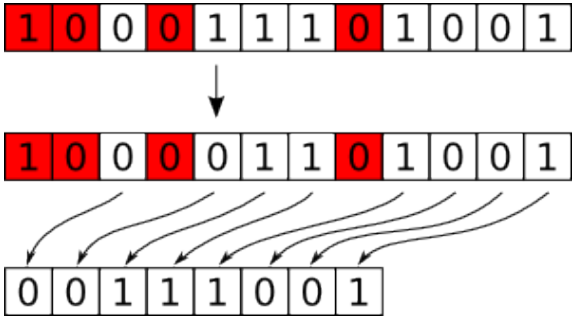

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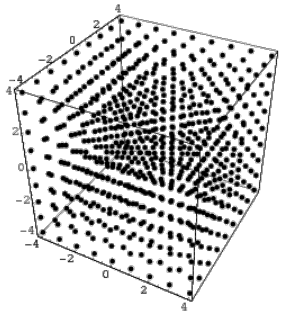

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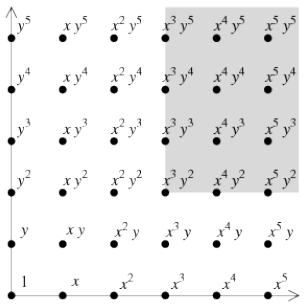

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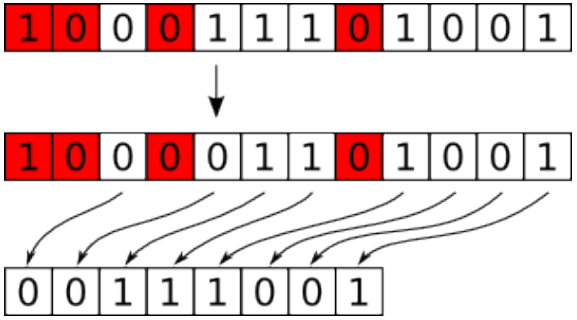

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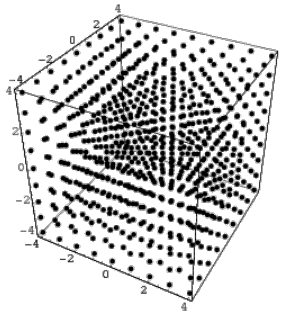

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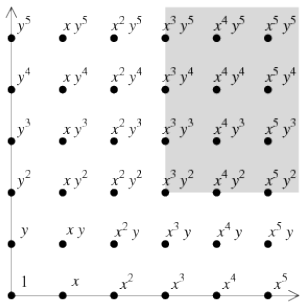

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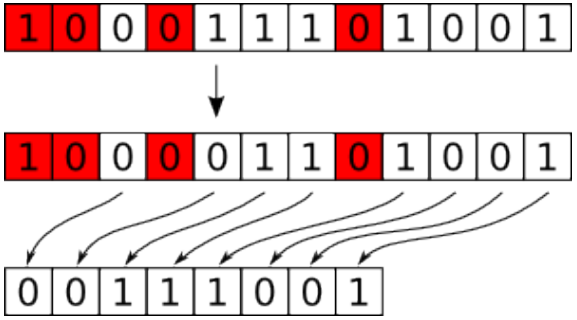

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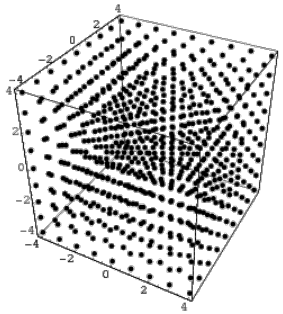

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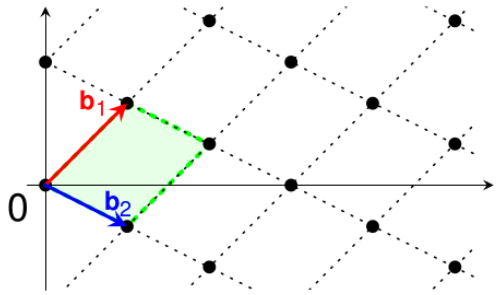

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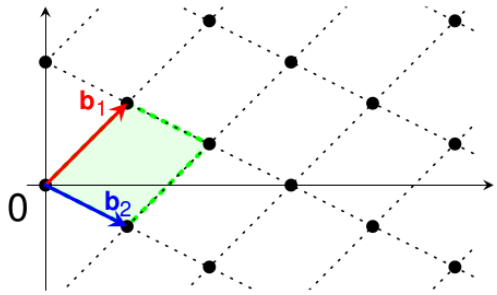

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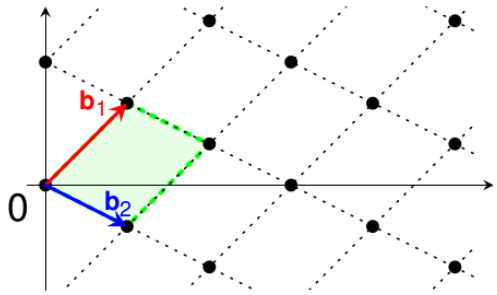

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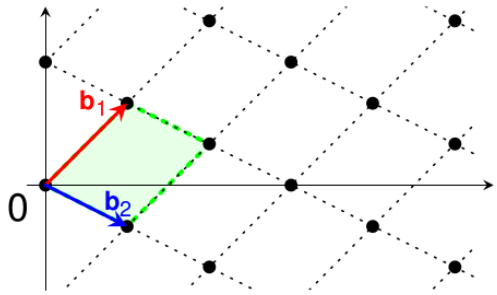

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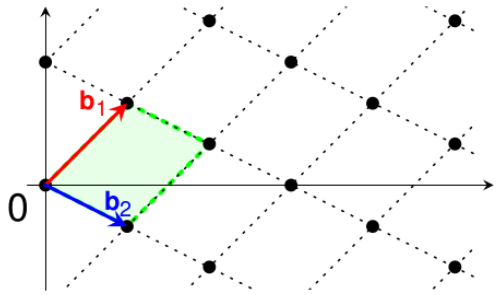

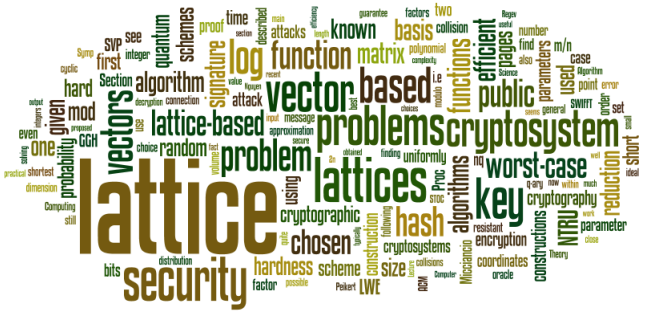

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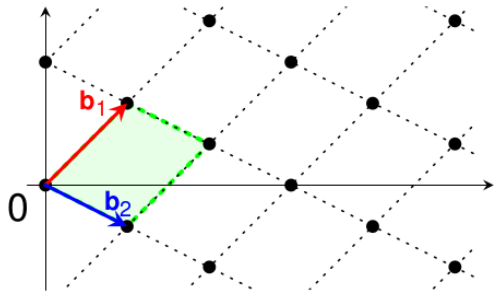

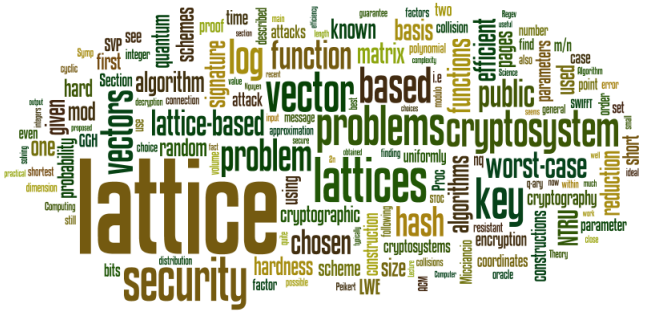

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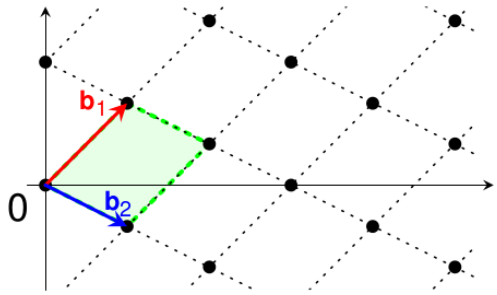

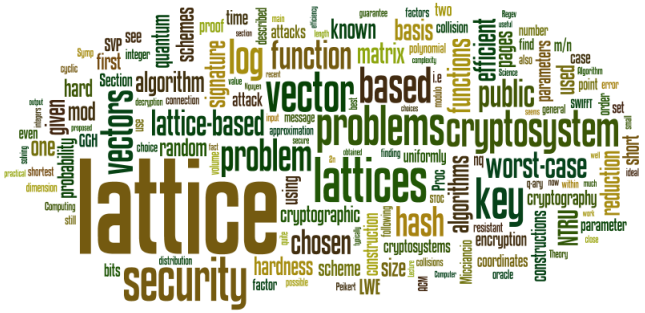

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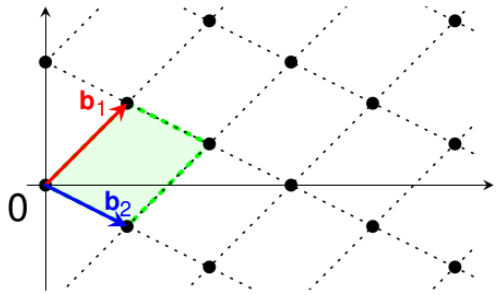
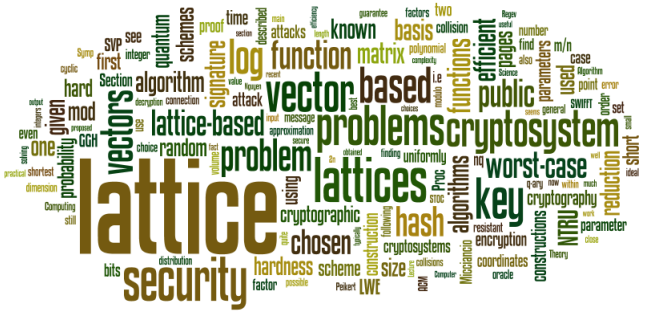
- ... and many more nice features!





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We proposed an efficent scheme by fixing a broken one using lattice techniques
[ABDG14] *Sealing the Leak on Classical NTRU Signatures*. Post-Quantum Cryptography 2014

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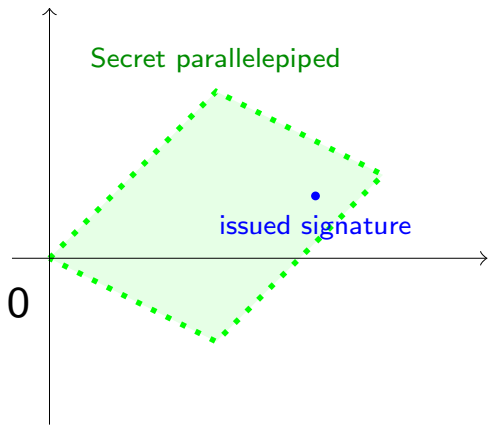

Number of signature: 1

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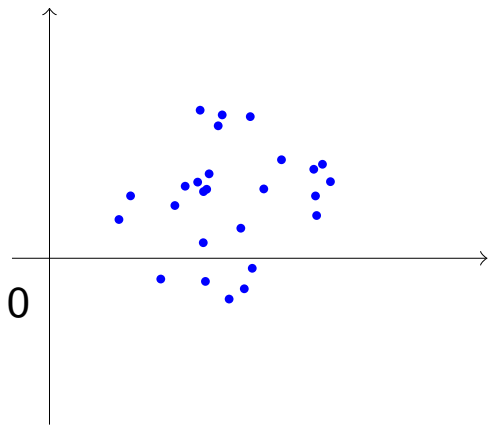

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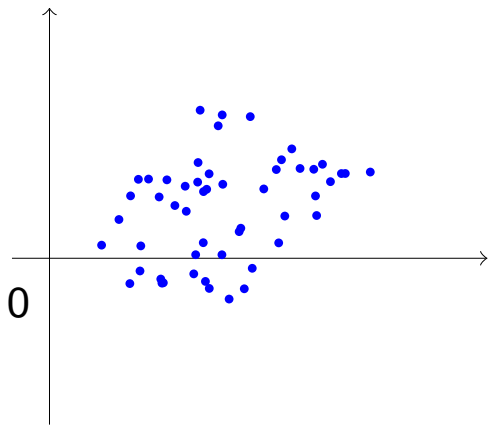

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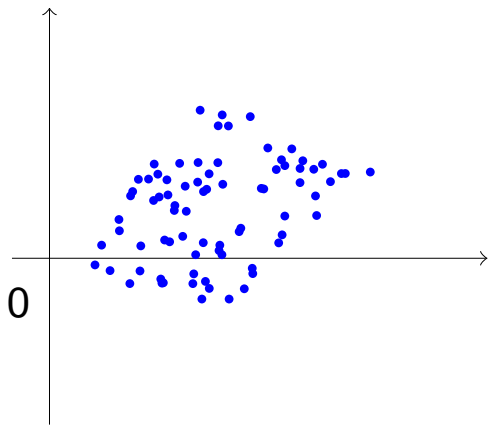

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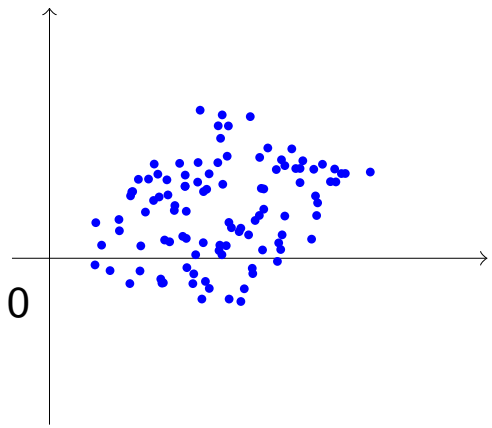

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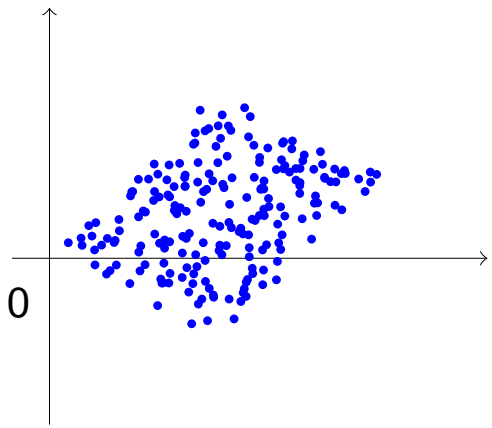
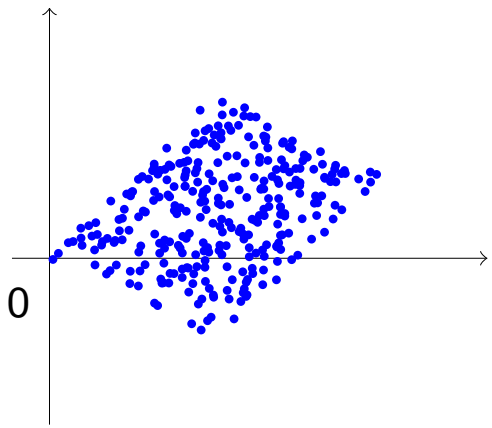


Number of signatures: 200

Results and Perspectives

Lattice-based Signature

We proposed an efficent scheme by fixing a broken one using lattice techniques
[ABDG14] *Sealing the Leak on Classical NTRU Signatures*. Post-Quantum Cryptography 2014



Number of signatures: 300

Results and Perspectives

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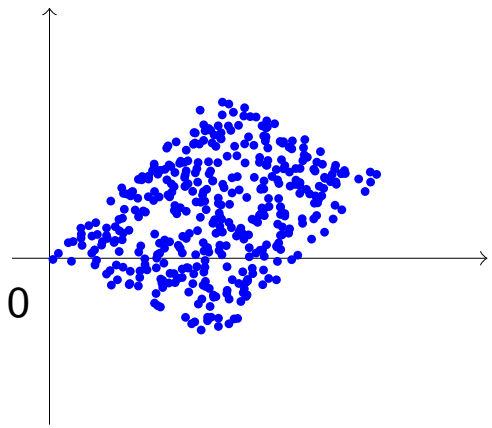

Number of signatures: 400

Results and Perspectives

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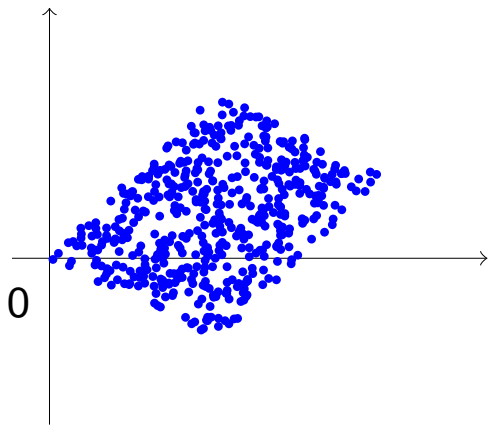
[ABDG14] *Sealing the Leak on Classical NTRU Signatures*. Post-Quantum Cryptography 2014



Number of signatures: 500

Results and Perspectives

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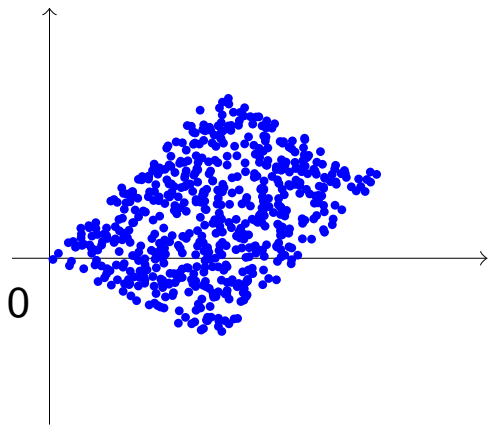

Number of signatures: 600

Results and Perspectives

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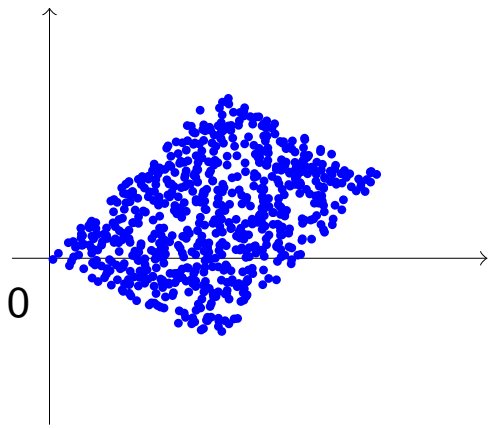
[ABDG14] *Sealing the Leak on Classical NTRU Signatures*. Post-Quantum Cryptography 2014



Number of signatures: 700

Results and Perspectives

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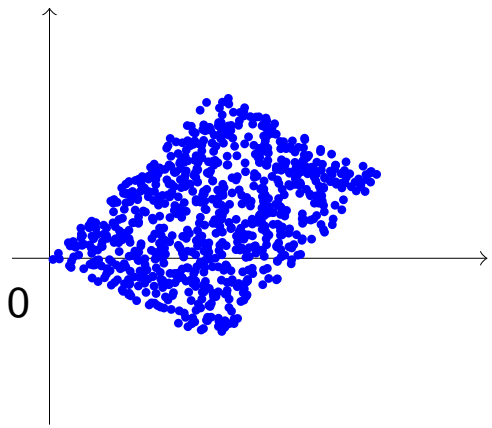

Number of signatures: 800

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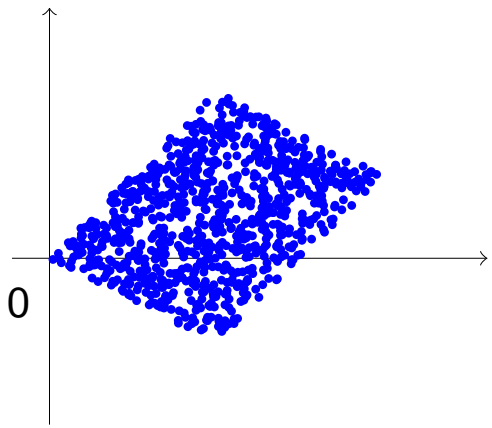

Number of signatures: 900

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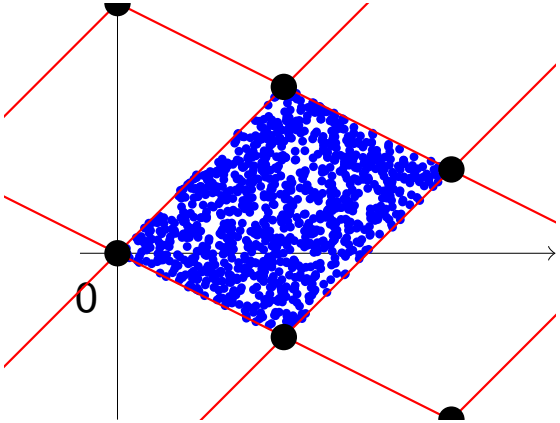

Number of signatures: 1000

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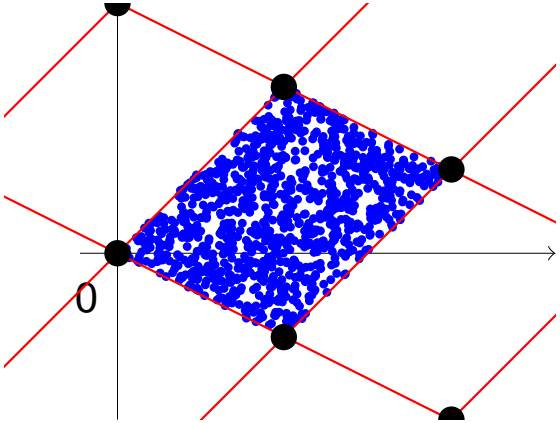

Number of signatures: 1000

Solution:
Rejection Sampling [Lyu12]

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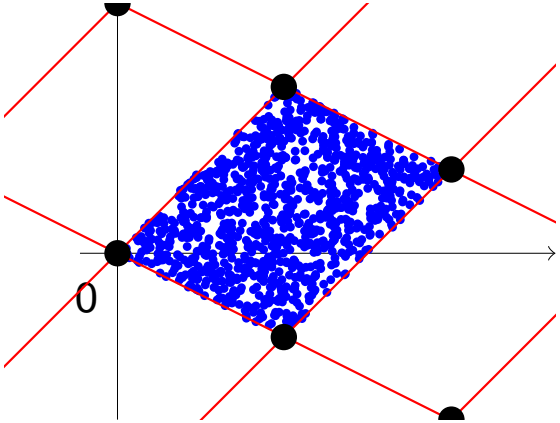

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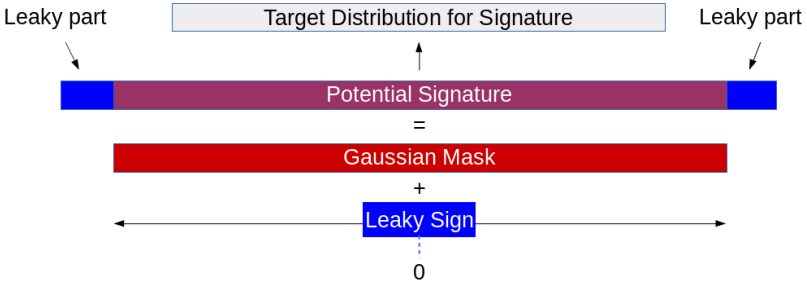

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Code-based Signature

We improved the efficiency of a class of signatures (namely threshold ring ones)
[DS14-15] *Improved Code-based Threshold Ring Signature Scheme*. Ongoing work

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Fully Homomorphic Encryption

Ongoing work in order to improve praticality...

References

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[Shor94] Shor, P. W. *Algorithms for Quantum Computation: Discrete Logarithms and Factoring*. FOCS'94

Thank You !