### Computing exponential solutions of a first order differential $Y'(x)=A(x)Y(x)$, using simple forms to compute the different exponential parts

An exponential solution has the form $\exp(\int(u(x)))P(x)$, where $u(x)$ is called an exponential part and $P(x)$ is a matrix with polynomial entries in $x$

#### Example from the paper of Pfulgel 1997

```
A := Matrix(3,3,[
  (-4 + x + x^2)/(x-1)*x^2, 4/(x-1)*x^2, (-4 + 2*x)/(x-1)*x^2,
  (4 - 3*x + 3*x^2)/(x-1)*x^2, (2*x+1)/(x-1)*x, (-3 + x)/(x-1)*x^2,
  (x^2 + x^2)/(x-1)*x^2, 2*x + 1/(x-1)*x^2, (x^2 + 1 + x)/(x-1)*x^2
])
```

(1)

This procedure computes exponential solutions. The output is of the form $[u_1, P_1, ..., u_m, P_m]$ where $u_i$ is an exponential part and $P_i$ is the corresponding polynomials

```
Sol := Exp_Sols(A, x);
```

(2)

This procedure computes a matrix $S$ whose columns form a basis of all exponential solutions. The output is the matrix $S$ and a list of all exponential (or hypergeometric) terms

```
SOL := ExponentialSolutions(A, x)
```
\[
SOL := \begin{bmatrix}
-\frac{1}{x} x^2 \\
-2 e^{-x} x^2 \\
\frac{1}{x} \\
2 e^{-x} x \\
\end{bmatrix}
\begin{bmatrix}
-\frac{1}{x} x^2 \\
\frac{1}{x} \\
-\frac{1}{x} x^2 \\
\frac{1}{x} \\
\end{bmatrix},
\begin{bmatrix}
e^{-x} x \\
\frac{x}{x-1}
\end{bmatrix}
\]

(3)

> check := simplify(map(diff, SOL[1], x) - A \cdot SOL[1]);

> check := \[
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]

(4)