Computing exponential solutions of a first order differential $Y'(x) = A(x)Y(x)$ (containing a parameter), using simple forms to compute the different exponential parts.

An exponential solution has the form $\exp(int(u(x))).P(x)$, where $u(x)$ is called an exponential part and $P(x)$ is a matrix with polynomial entries in $x$.

We load our package

with(PseudoLinearSystems);

### Example 5 of Bronstien, Li, Wu 2005

$A := \begin{pmatrix}
\frac{(x + 1)}{k}, & \frac{k \cdot (x + 1 - k)}{x^2 \cdot (k - 1)}, & -\frac{k \cdot (x + 1 - k)}{x^2 \cdot (k - 1)}, & x + 1,
\frac{(x \cdot k - k^2 + 2 \cdot x^2 + k \cdot x^2 + k - 1)}{x \cdot (k - 1)}, & -\frac{x \cdot k - k^2 + 2 \cdot x^2 + k \cdot x^2}{x \cdot (k - 1)}, & x + 1,
\frac{(x \cdot k + 2 \cdot x^2 + k \cdot x^2 - 2 \cdot k^2 + k)}{x \cdot (k - 1)}, & -\frac{x \cdot k + 2 \cdot x^2 + k \cdot x^2 - 2 \cdot k^2 + 1}{x \cdot (k - 1)}
\end{pmatrix}$

This procedure computes exponential solutions. The output is of the form $[u_1, P_1, ..., u_m, P_m]$ where $u_i$ is an exponential part and $P_i$ is the corresponding polynomials.

$Sol := \text{Exp_Sols}(A, x)$;

$Sol := \begin{bmatrix}
0, & -k x \n-k x
\end{bmatrix}$

This procedure computes a matrix $S$ whose columns form a basis of all exponential solutions. The output is the matrix $S$ and a list of all exponential (or hypergeometric) terms.

$SOL := \text{ExponentialSolutions}(A, x)$

$SOL := \begin{bmatrix}
0
-k x
\end{bmatrix}$
No exponential solutions, just polynomial solutions

\[ \text{check} := \text{simplify}(\text{map(}\text{diff, SOL[1], x}) - A \cdot \text{SOL[1]}) \]