

This example file illustrates the main procedures of the package StabilityEquivalence on an example.

```
> restart;
```

To start the package, just read the .mpl file.

```
> read "/Users/cluzeau/Documents/Projects/ANR-MSDOS/StabiliteViaEquivalence/Maple/StabilityEquivalence.mpl":
```

I - Generalized Fornasini-Marchesini (FM) systems and structural stability

Let us define the dimensions (dx of the state variable x and du of the input variable u) and the matrices of a **generalized Fornasini-Marchesini (FM) system** of the form

$$(i,j) \quad x(i+1,j+1) = F1*x(i+1,j)+F2*x(i,j+1)+F3*x(i,j)+G1*u(i+1,j)+G2*u(i,j+1)+G3*u$$

Remark: we are going to use computer algebra techniques which can only be performed over an **effective field** so that the entries of your matrices must have **rational (and not real) coefficients!**

```
> dx:=3;
   du:=1;
```

```
dx := 3
```

```
du := 1
```

(1.1)

```
> F1 := convert(Matrix(dx,dx,[0.7815, 0.8189,0.1054,0.3428,0.2393,
0.2070,0.0480,0.0369,0.3624]),rational);
F2 := convert(Matrix(dx,dx,[0.3652, 0.3623,0.0537,0.0596,0.6569,
0.1596,0.3674,0.3029,0.8198]),rational);
F3 := convert(Matrix(dx,dx,[0.5008,0.0225,1.2194,0.6066,1.3271,
0.4790,0.4145,0.5905,0.4698]),rational);
G1 := convert(Matrix(dx,du,[0.3922,0.6555,0.1712]),rational);
G2 := convert(Matrix(dx,du,[0.7060,0.0318,0.2769]),rational);
G3 := convert(Matrix(dx,du,[0.0462,0.0971,0.8235]),rational);
```

$$F1 := \begin{bmatrix} \frac{1563}{2000} & \frac{8189}{10000} & \frac{527}{5000} \\ \frac{857}{2500} & \frac{2393}{10000} & \frac{207}{1000} \\ \frac{6}{125} & \frac{369}{10000} & \frac{453}{1250} \end{bmatrix}$$

$$\begin{aligned}
 F2 &:= \begin{bmatrix} \frac{913}{2500} & \frac{3623}{10000} & \frac{537}{10000} \\ \frac{149}{2500} & \frac{6569}{10000} & \frac{399}{2500} \\ \frac{1837}{5000} & \frac{3029}{10000} & \frac{4099}{5000} \end{bmatrix} \\
 F3 &:= \begin{bmatrix} \frac{313}{625} & \frac{9}{400} & \frac{6097}{5000} \\ \frac{3033}{5000} & \frac{13271}{10000} & \frac{479}{1000} \\ \frac{829}{2000} & \frac{1181}{2000} & \frac{2349}{5000} \end{bmatrix} \\
 G1 &:= \begin{bmatrix} \frac{1961}{5000} \\ \frac{1311}{2000} \\ \frac{107}{625} \end{bmatrix} \\
 G2 &:= \begin{bmatrix} \frac{353}{500} \\ \frac{159}{5000} \\ \frac{2769}{10000} \end{bmatrix} \\
 G3 &:= \begin{bmatrix} \frac{231}{5000} \\ \frac{971}{10000} \\ \frac{1647}{2000} \end{bmatrix}
 \end{aligned} \tag{1.2}$$

We can now use the procedure *TestStructuralStabilityFornasini2D* to test whether the latter (FM) model is structurally stable or not. Here we use the package **IsStable**

```

> TestStructuralStabilityFornasini2D(F1,F2,F3,dx);
false

```

(1.3)

Here we see that our (FM) model is not structurally stable.

II - Transformation into an equivalent Roesser (R) model

We can now convert our (FM) model to an equivalent **Roesser (R)** model of the form

$$\begin{pmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{pmatrix} = A \begin{pmatrix} x^h(i,j) \\ x^v(i,j) \end{pmatrix} + B u'(i,j)$$

where $dh(=dx)$ is the dimension of the horizontal state variable x^h and $dv(=dx+du)$ is the dimension of the vertical state variable x^v

For this we call the procedure *FornasiniToRoesser2D*

> A, B, dh, dv := FornasiniToRoesser2D(F1, F2, F3, G1, G2, G3, dx);

A, B, dh,

(2.1)

$$dv := \left[\left[\frac{913}{2500}, \frac{3623}{10000}, \frac{537}{10000}, \frac{11412223}{12500000}, \frac{2051211}{5000000}, \frac{67617453}{50000000} \right], \right. \\ \left. \frac{43611253}{100000000} \right], \\ \left[\frac{149}{2500}, \frac{6569}{10000}, \frac{399}{2500}, \frac{5537647}{6250000}, \frac{30779837}{20000000}, \frac{33954959}{50000000}, \frac{57839659}{100000000} \right], \\ \left[\frac{1837}{5000}, \frac{3029}{10000}, \frac{4099}{5000}, \frac{42240381}{50000000}, \frac{19881969}{20000000}, \frac{43415989}{50000000}, \frac{130649499}{100000000} \right], \\ \left[1, 0, 0, \frac{1563}{2000}, \frac{8189}{10000}, \frac{527}{5000}, \frac{1961}{5000} \right], \\ \left[0, 1, 0, \frac{857}{2500}, \frac{2393}{10000}, \frac{207}{1000}, \frac{1311}{2000} \right], \\ \left[0, 0, 1, \frac{6}{125}, \frac{369}{10000}, \frac{453}{1250}, \frac{107}{625} \right], \\ \left[0, 0, 0, 0, 0, 0, 0 \right], \left[\frac{353}{500}, \frac{159}{5000}, \frac{2769}{10000}, 0, 0, 0, 1 \right], 3, 4$$

We can now use the procedure *TestStructuralStabilityRoesser2D* to test if the latter (R) model is structurally stable.

```
> TestStructuralStabilityRoesser2D(A, dh, dv);
false (2.2)
```

Of course, as it is equivalent to our original (FM) model, we find that the latter (R) model is not structurally stable.

III - Equivalence of Systems (optional)

Here we use the packages **OreModules** (<http://wwwb.math.rwth-aachen.de/OreModules/>) and **OreMorphisms** (http://www.unilim.fr/pages_perso/thomas.cluzeau/Packages/OreMorphisms/index.html) that can be freely downloaded to check again that the latter (FM) and (R) systems are equivalent in the sense of algebraic analysis. See our related papers for more details.

Note that this section just checks the isomorphism and can be skipped. In this case, one would use the explicit formula in our paper to construct the matrix Pp below which will be useful in section V for the stabilization.

```
> with(OreModules);
> with(OreMorphisms);
> Alg:=DefineOreAlgebra(diff=[sigma1,i],diff=[sigma2,j],polynom=[i,
j],comm=[]):
```

We first need to construct the matrices of operators associated to our models using the procedures *FornasiniToOperatorMatrix2D* and *RoesserToOperatorMatrix2D*:

```
> R_FM := FornasiniToOperatorMatrix2D(F1,F2,F3,G1,G2,G3,dx);
R_FM := [[ [sigma1 sigma2 - 1563/2000 sigma1 - 913/2500 sigma2 - 313/625, - 8189/10000 sigma1 - 3623/10000 sigma2 - 9/400, (3.1)
- 527/5000 sigma1 - 537/10000 sigma2 - 6097/5000, - 1961/5000 sigma1 - 353/500 sigma2 - 231/5000 ],
[ - 857/2500 sigma1 - 149/2500 sigma2 - 3033/5000, sigma1 sigma2 - 2393/10000 sigma1 - 6569/10000 sigma2 - 13271/10000,
- 207/1000 sigma1 - 399/2500 sigma2 - 479/1000, - 1311/2000 sigma1 - 159/5000 sigma2 - 971/10000 ],
[ - 6/125 sigma1 - 1837/5000 sigma2 - 829/2000, - 369/10000 sigma1 - 3029/10000 sigma2 - 1181/2000, sigma1 sigma2
- 453/1250 sigma1 - 4099/5000 sigma2 - 2349/5000, - 107/625 sigma1 - 2769/10000 sigma2 - 1647/2000 ] ]]
```

```
> R_Ro := RoesserToOperatorMatrix2D(A,B,dh,dv);
R_Ro := [[ [sigma1 - 913/2500, - 3623/10000, - 537/10000, - 11412223/12500000, - 2051211/5000000, - 67617453/50000000, (3.2)
- 43611253/100000000, - 353/500 ],
```

$$\begin{aligned}
& \left[-\frac{149}{2500}, \sigma I - \frac{6569}{10000}, -\frac{399}{2500}, -\frac{5537647}{6250000}, -\frac{30779837}{20000000}, -\frac{33954959}{50000000}, \right. \\
& \left. -\frac{57839659}{100000000}, -\frac{159}{5000} \right], \\
& \left[-\frac{1837}{5000}, -\frac{3029}{10000}, \sigma I - \frac{4099}{5000}, -\frac{42240381}{50000000}, -\frac{19881969}{20000000}, -\frac{43415989}{50000000}, \right. \\
& \left. -\frac{130649499}{100000000}, -\frac{2769}{10000} \right], \\
& \left[-1, 0, 0, \sigma^2 - \frac{1563}{2000}, -\frac{8189}{10000}, -\frac{527}{5000}, -\frac{1961}{5000}, 0 \right], \\
& \left[0, -1, 0, -\frac{857}{2500}, \sigma^2 - \frac{2393}{10000}, -\frac{207}{1000}, -\frac{1311}{2000}, 0 \right], \\
& \left[0, 0, -1, -\frac{6}{125}, -\frac{369}{10000}, \sigma^2 - \frac{453}{1250}, -\frac{107}{625}, 0 \right], \\
& \left[0, 0, 0, 0, 0, 0, \sigma^2, -1 \right]
\end{aligned}$$

The matrix P defining the isomorphism between the two models is the following (see our related papers):

```

> P := convert(linalg[stackmatrix](linalg[augment](Matrix(dx,dx),
LinearAlgebra[IdentityMatrix](dx),Matrix(dx,du),Matrix(dx,du)),
linalg[augment](Matrix(du,dx),Matrix(du,dx),LinearAlgebra
[IdentityMatrix](du),Matrix(du,du))),Matrix);

```

$$P := \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (3.3)$$

We now use **OreMorphisms** to check that it define a morphism

```

> Q:=Factorize(Mult(R_FM,P,Alg),R_Ro,Alg);

```

$$Q := \begin{bmatrix} 1 & 0 & 0 & \sigma I - \frac{913}{2500} & -\frac{3623}{10000} & -\frac{537}{10000} & -\frac{353}{500} \\ 0 & 1 & 0 & -\frac{149}{2500} & \sigma I - \frac{6569}{10000} & -\frac{399}{2500} & -\frac{159}{5000} \\ 0 & 0 & 1 & -\frac{1837}{5000} & -\frac{3029}{10000} & \sigma I - \frac{4099}{5000} & -\frac{2769}{10000} \end{bmatrix} \quad (3.4)$$

and then check that it is an isomorphism

```

> TestIso(R_FM,R_Ro,P,Alg);

```

true (3.5)

whose inverse is given by

```

> invMorph := InverseMorphism(R_FM,R_Ro,P,Alg);

```

$$\text{invMorph} := \left[\begin{array}{cccc|cccc}
 \sigma^2 - \frac{1563}{2000} & -\frac{8189}{10000} & -\frac{527}{5000} & -\frac{1961}{5000} & 1 & 0 & 0 & \\
 -\frac{857}{2500} & \sigma^2 - \frac{2393}{10000} & -\frac{207}{1000} & -\frac{1311}{2000} & 0 & 1 & 0 & \\
 -\frac{6}{125} & -\frac{369}{10000} & \sigma^2 - \frac{453}{1250} & -\frac{107}{625} & 0 & 0 & 1 & \\
 1 & 0 & 0 & 0 & 0 & 0 & 0 & \\
 0 & 1 & 0 & 0 & 0 & 0 & 0 & \\
 0 & 0 & 1 & 0 & 0 & 0 & 0 & \\
 0 & 0 & 0 & 1 & 0 & 0 & 0 & \\
 0 & 0 & 0 & 1 & 0 & 0 & 0 & \\
 0 & 0 & 0 & \sigma^2 & 0 & 0 & 0 &
 \end{array} \right], \quad (3.6)$$

```

> Pp:=convert(invMorph[1],Matrix);
Qp:=invMorph[2];

```

$$Pp := \left[\begin{array}{cccc}
 \sigma^2 - \frac{1563}{2000} & -\frac{8189}{10000} & -\frac{527}{5000} & -\frac{1961}{5000} \\
 -\frac{857}{2500} & \sigma^2 - \frac{2393}{10000} & -\frac{207}{1000} & -\frac{1311}{2000} \\
 -\frac{6}{125} & -\frac{369}{10000} & \sigma^2 - \frac{453}{1250} & -\frac{107}{625} \\
 1 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 \\
 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 1 \\
 0 & 0 & 0 & \sigma^2
 \end{array} \right]$$

$$Qp := \left[\begin{array}{ccc}
 1 & 0 & 0 \\
 0 & 1 & 0 \\
 0 & 0 & 1 \\
 0 & 0 & 0 \\
 0 & 0 & 0 \\
 0 & 0 & 0 \\
 0 & 0 & 0
 \end{array} \right] \quad (3.7)$$

Finally we can check that the needed equalities are fulfilled

```

> verif1:=convert(Mult(R_FM,P,Alg)-Mult(Q,R_Ro,Alg),set);
verif2:=convert(Mult(R_Ro,Pp,Alg)-Mult(Qp,R_FM,Alg),set);
Z:=Factorize(evalm(1-Mult(P,Pp,Alg)),R_FM,Alg);
Zp:=Factorize(evalm(1-Mult(Pp,P,Alg)),R_Ro,Alg);
verif1 := {0}

```

$$\begin{aligned}
& \text{verif2} := \{0\} \\
& Z := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
& Z_p := \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{3.8}
\end{aligned}$$

IV - Structural Stabilisation of the R model

Using a recent result of Bachelier et al (see *O. Bachelier, N. Yeganefar, D. Mehdi, and W. Paszke. State feedback structural stabilization of 2D discrete Roesser models. Proceedings of the 9th International Workshop on Multidimensional (nD) Systems (nDS'15), Vila Real (Portugal), 07-09/09/2015*), we can find a stabilizing state feedback control law of the form $u' = K1*x^h + K2*x^v$ for the (R) model. Here we only take the result obtained using a solver of Linear Matrix Inequalities (LMIs) in Matlab. The problem of computing a stabilizing state feedback control law for a Roesser model is not handled by the package **StabilityEquivalence**.

```

> K1 := map(convert, Matrix(1,3, [-0.9469, -1.1764, -0.8413]), rational)
;
K2 := map(convert, Matrix(1,4, [-1.3196, -1.2745, -0.8113, -1.4124]),
rational);
K := convert(linalg[augment](K1, K2), Matrix);

```

$$\begin{aligned}
& K1 := \begin{bmatrix} -\frac{9469}{10000} & -\frac{2941}{2500} & -\frac{8413}{10000} \end{bmatrix} \\
& K2 := \begin{bmatrix} -\frac{3299}{2500} & -\frac{2549}{2000} & -\frac{8113}{10000} & -\frac{3531}{2500} \end{bmatrix} \\
& K := \begin{bmatrix} -\frac{9469}{10000} & -\frac{2941}{2500} & -\frac{8413}{10000} & -\frac{3299}{2500} & -\frac{2549}{2000} & -\frac{8113}{10000} & -\frac{3531}{2500} \end{bmatrix} \tag{4.1}
\end{aligned}$$

Applying the stabilizing state feedback control law $u' = K1*x^h + K2*x^v$ to the latter (R) model, we obtain the "input-free" (autonomous) (R) model given by the following matrix

```

> As := A + B. K ;

```

(4.2)

$$\begin{aligned}
 A_s := & \left[\left[\left[-\frac{1516557}{5000000}, -\frac{292649}{625000}, -\frac{2701289}{5000000}, -\frac{233247}{12500000}, -\frac{1223887}{2500000}, \frac{38978563}{50000000}, \right. \right. \right. \\
 & \left. \left. \left. -\frac{56104187}{100000000} \right], \right. \right. \\
 & \left[\frac{1474429}{50000000}, \frac{7743631}{12500000}, \frac{6642333}{50000000}, \frac{10550753}{12500000}, \frac{5993851}{4000000}, \frac{1020781}{1562500}, \frac{53348227}{100000000} \right. \\
 & \left. \left. \left. \left[\frac{10520339}{100000000}, -\frac{571129}{25000000}, \frac{58684403}{100000000}, \frac{23970519}{50000000}, \frac{3205947}{5000000}, \frac{64367081}{100000000}, \right. \right. \right. \\
 & \left. \left. \left. \frac{91540143}{100000000} \right], \right. \right. \\
 & \left[1, 0, 0, \frac{1563}{2000}, \frac{8189}{10000}, \frac{527}{5000}, \frac{1961}{5000} \right], \\
 & \left[0, 1, 0, \frac{857}{2500}, \frac{2393}{10000}, \frac{207}{1000}, \frac{1311}{2000} \right], \\
 & \left[0, 0, 1, \frac{6}{125}, \frac{369}{10000}, \frac{453}{1250}, \frac{107}{625} \right], \\
 & \left. \left. \left. \left[-\frac{9469}{10000}, -\frac{2941}{2500}, -\frac{8413}{10000}, -\frac{3299}{2500}, -\frac{2549}{2000}, -\frac{8113}{10000}, -\frac{3531}{2500} \right] \right] \right]
 \end{aligned} \tag{4.2}$$

and we can check that this new "input-free" (autonomous) (R) model is structurally stable

```

> TestStructuralStabilityRoesser2D(As, dh, dv);
true
\tag{4.3}

```

V - Effect on the original FM model

If we now apply to the original (FM) model the control law corresponding to applying the state feedback control law $u = K_1 * x^h + K_2 * x^v$ to the equivalent (R) model, we get the linear system given by the matrix of operators FMs above (for more detail see our related papers)

To achieve this, we need the matrix Pp computed in Section III. If you have skipped section III, then you can use the following formula

$$Pp1 := \text{convert}(\text{linalg}[\text{stackmatrix}](\text{linalg}[\text{augment}](\text{sigma2} * \text{LinearAlgebra}[\text{IdentityMatrix}](dx) - F1, -G1), \text{linalg}[\text{augment}](\text{LinearAlgebra}[\text{IdentityMatrix}](dx), \text{Matrix}(dx, du)), \text{linalg}[\text{augment}](\text{Matrix}(du, dx), \text{LinearAlgebra}[\text{IdentityMatrix}](du)), \text{linalg}[\text{augment}](\text{Matrix}(du, dx), \text{sigma2} * \text{LinearAlgebra}[\text{IdentityMatrix}](du))), \text{Matrix});$$

to get Pp. Here we check that it provides the same matrix.

```

> Pp1 := convert(linalg[stackmatrix](linalg[augment](sigma2 *

```



```

LinearAlgebra[IdentityMatrix](dx)-F1,-G1),linalg[augment]
(LinearAlgebra[IdentityMatrix](dx),Matrix(dx,du)),linalg[augment]
(Matrix(du,dx),LinearAlgebra[IdentityMatrix](du)),linalg[augment]
(Matrix(du,dx),sigma2*LinearAlgebra[IdentityMatrix](du)),Matrix
;

```

$$Pp1 := \begin{bmatrix} \sigma^2 - \frac{1563}{2000} & -\frac{8189}{10000} & -\frac{527}{5000} & -\frac{1961}{5000} \\ -\frac{857}{2500} & \sigma^2 - \frac{2393}{10000} & -\frac{207}{1000} & -\frac{1311}{2000} \\ -\frac{6}{125} & -\frac{369}{10000} & \sigma^2 - \frac{453}{1250} & -\frac{107}{625} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \sigma^2 \end{bmatrix} \quad (5.1)$$

```

> convert(simplify(Pp-Pp1),set);
{0}
(5.2)

```

We now get the need submatrices of Pp:

```

> Pp11 := LinearAlgebra[SubMatrix](Pp,1..(2*dx+du),1..dx);
Pp12 := LinearAlgebra[SubMatrix](Pp,1..(2*dx+du),dx+1..dx+du);
Pp21 := LinearAlgebra[SubMatrix](Pp,(2*dx+du)+1..(2*dx+2*du),1..
dx);
Pp22 := LinearAlgebra[SubMatrix](Pp,(2*dx+du)+1..(2*dx+2*du),
dx+1..dx+du);

```

$$Pp11 := \begin{bmatrix} \sigma^2 - \frac{1563}{2000} & -\frac{8189}{10000} & -\frac{527}{5000} \\ -\frac{857}{2500} & \sigma^2 - \frac{2393}{10000} & -\frac{207}{1000} \\ -\frac{6}{125} & -\frac{369}{10000} & \sigma^2 - \frac{453}{1250} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned}
 Pp12 &:= \begin{bmatrix} -\frac{1961}{5000} \\ -\frac{1311}{2000} \\ -\frac{107}{625} \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \\
 Pp21 &:= \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \\
 Pp22 &:= \begin{bmatrix} \sigma_2 \end{bmatrix}
 \end{aligned} \tag{5.3}$$

> FMs := convert(linalg[stackmatrix](R_FM,linalg[augment](-K.Pp11+Pp21,-K.Pp12+Pp22)),Matrix);

$$\begin{aligned}
 FMs &:= \left[\left[\sigma_1 \sigma_2 - \frac{1563}{2000} \sigma_1 - \frac{913}{2500} \sigma_2 - \frac{313}{625}, -\frac{8189}{10000} \sigma_1 - \frac{3623}{10000} \sigma_2 - \frac{9}{400}, \right. \right. \\
 &\quad \left. \left. -\frac{527}{5000} \sigma_1 - \frac{537}{10000} \sigma_2 - \frac{6097}{5000}, -\frac{1961}{5000} \sigma_1 - \frac{353}{500} \sigma_2 - \frac{231}{5000} \right], \right. \\
 &\quad \left[-\frac{857}{2500} \sigma_1 - \frac{149}{2500} \sigma_2 - \frac{3033}{5000}, \sigma_1 \sigma_2 - \frac{2393}{10000} \sigma_1 - \frac{6569}{10000} \sigma_2 - \frac{13271}{10000}, \right. \\
 &\quad \left. -\frac{207}{1000} \sigma_1 - \frac{399}{2500} \sigma_2 - \frac{479}{1000}, -\frac{1311}{2000} \sigma_1 - \frac{159}{5000} \sigma_2 - \frac{971}{10000} \right], \\
 &\quad \left[-\frac{6}{125} \sigma_1 - \frac{1837}{5000} \sigma_2 - \frac{829}{2000}, -\frac{369}{10000} \sigma_1 - \frac{3029}{10000} \sigma_2 - \frac{1181}{2000}, \sigma_1 \sigma_2 \right. \\
 &\quad \left. -\frac{453}{1250} \sigma_1 - \frac{4099}{5000} \sigma_2 - \frac{2349}{5000}, -\frac{107}{625} \sigma_1 - \frac{2769}{10000} \sigma_2 - \frac{1647}{2000} \right], \\
 &\quad \left[\frac{9469}{10000} \sigma_2 + \frac{13594533}{100000000}, \frac{1865271}{10000000} + \frac{2941}{2500} \sigma_2, \frac{8154741}{50000000} + \frac{8413}{10000} \sigma_2, \right. \\
 &\quad \left. \frac{6293253}{50000000} + \sigma_2 \right] \tag{5.4}
 \end{aligned}$$

We can now check that this system is structurally stable using the procedure

TestStructuralStabilityAutonomousLinearSystem2D

> TestStructuralStabilityAutonomousLinearSystem2D(FMs);
true (5.5)

However, we can note that the latter system is not a FM model!

If one wants to find a (FM) model equivalent to the structurally stable (R) model that we have found, then one can try to use the *RoesserToFornasini2D* procedure

> FMs1 := RoesserToFornasini2D(As,Matrix(dh+dv,du),dh,dv);

$$FMs1 := \begin{bmatrix} \frac{1563}{2000} & \frac{8189}{10000} & \frac{527}{5000} & \frac{1961}{5000} \\ \frac{857}{2500} & \frac{2393}{10000} & \frac{207}{1000} & \frac{1311}{2000} \\ \frac{6}{125} & \frac{369}{10000} & \frac{453}{1250} & \frac{107}{625} \\ -\frac{3299}{2500} & -\frac{2549}{2000} & -\frac{8113}{10000} & -\frac{3531}{2500} \end{bmatrix}, \quad (5.6)$$

$$\begin{bmatrix} 0 & -\frac{509173}{5570000} & -\frac{25639423}{94690000} & \frac{1516557}{4734500} \\ 0 & \frac{3246501}{5570000} & \frac{1262297}{11836250} & -\frac{1474429}{47345000} \\ 0 & -\frac{171051}{1114000} & \frac{18687}{37876} & -\frac{10520339}{94690000} \\ 0 & -\frac{3271902289}{6962500000} & -\frac{269051926377}{946900000000} & -\frac{164007587141}{946900000000} \end{bmatrix}, \left[\begin{array}{l} \left[\frac{106140071371}{236725000000}, -\frac{1170361173}{23672500000}, \frac{136885953967}{118362500000}, -\frac{277392489}{118362500000} \right], \\ \left[\frac{141571799583}{236725000000}, \frac{31137849371}{236725000000}, \frac{55480581091}{118362500000}, \frac{10555304053}{118362500000} \right], \\ \left[\frac{171271867879}{473450000000}, \frac{24530719673}{473450000000}, \frac{96233145783}{236725000000}, \frac{183382331739}{236725000000} \right], \\ \left[-\frac{6781892196678849}{4734500000000000}, -\frac{916823964715563}{4734500000000000}, -\frac{4707302763608473}{2367250000000000} \right], \\ -\frac{1785887491722509}{2367250000000000} \right], \left[\begin{array}{l} \left[0 \right] \\ \left[0 \right] \\ \left[0 \right] \\ \left[0 \right] \end{array} \right]$$

and we can also test that the obtained (FM) model is structurally stable.

```
> TestStructuralStabilityFornasini2D(FMs1[1], FMs1[2], FMs1[3],
LinearAlgebra[RowDimension](FMs1[1]));
true
```

(5.7)