```
> restart:
```

We load our package:

```
> with(IntegrableConnections);
```

[*Eigenring*, *Evalm*, *G_matrix*, *G_matrix0*, *HyperexponentialSolutions*, *Id*, *Mapply*, *MatrixColumnPrimpart*, *MatrixColumnPrimpartRat*, *Mexpsolde*, *Mpolsolde*, *Mratsolde*, *Msylvester*, *Mylinsolve*, *NNIexponents*, *PolynomialSolutions*, *RationalSolutions*, *Reduction*, *TestIntegrabilityConditions*, *Trigonalize*, *backsubs*, *candidates*, *col_pval*, *colechelon*, *colval*, *complement*, *convert_back*, *denomatrix*, *denomvector*, *direct_ratsol*, *eval_block_reduced*, *evalam*, *exp_parts*, *formal_reduce*, *gen_sylvester*, *getargs*, *good_form*, *intDiff*, *int_diff_split*, *l_colechelon*, *l_column_rank*, *l_moser_reduce*, *l_qtcd*, *l_super*, *l_super_reduce*, *lc_evala*, *lc_evalam*, *localize*, *log_collect*, *log_free*, *mat_block_reduce*, *mat_change_exp_part*, *mat_check*, *mat_coeff*, *mat_colvaluation*, *mat_convert*, *mat_copy*, *mat_difference*, *mat_eval*, *mat_get_dim*, *mat_get_exp_part*, *mat_get_ext*, *mat_get_indicial_polynomial*, *mat_get_inv_transformation*, *mat_get_order*, *mat_get_ramification*, *mat_get_terms*, *mat_get_transformation*, *mat_get_type*, *mat_lc*, *mat_lead_mon*, *mat_product*, *mat_pval*, *mat_subs*, *mat_sum*, *mat_swap*, *mat_to_simple*, *mat_tools*, *mat_transform*, *mat_val*, *mat_valuation*, *matrix_series*, *matval*, *minimalIntegerRoot*, *mult*, *my_linsolve*, *mydegree*, *myequaind*, *new_matrix*, *newton*, *newton_polygon*, *nonzero*, *normalm*, *pencil_block_reduce*, *pencil_solve*, *pgaussUnimod*, *poly_pval*, *poly_sols*, *poly_val*, *prank*, *pval*, *qtcd*, *ramified_case*, *ramified_reduction*, *randomMatrix*, *randomPoly*, *randomRat*, *ratsuper*, *reduceColRank4*, *reg_sols*, *regular_series*, *remove*, *rowechelon*, *seq_evala*, *sim_sylvester*, *simple_eval*, *simple_sols*, *sortColumns*, *super_form*, *system_expsolde*, *tensor_prod*, *transform*, *truncate*, *utils*, *val*, *val1*, *val_ldegree*, *valuation*, *vectdegree*] **(1)**

The library linalg is also needed.

```
> with(linalg):
```

We load OreModules in order to use the procedure to write a D-finite partial differential system to an integrable connection.

```
> with(OreModules):
```

BrycLetac system in dimension 2 - Gaussian Case

We define the OreAlgebra (needed for OreModules):

```
> Alg2:=DefineOreAlgebra(diff=[d1,x1],diff=[d2,x2],polynom=[x1,x2],
  comm=[d,a]):
```

We give the equations of the system:

```
> R2 := matrix(2,1,[-(1/2)*d*d2+d1^2-x2*d2^2,2*d1*d2+x1*d2^2]);
```

$$R2 := \begin{bmatrix} -\frac{1}{2}\, d\, d2 + d1^2 - x2\, d2^2 \\ 2\, d1\, d2 + x1\, d2^2 \end{bmatrix} \quad \textbf{(2)}$$

We write the system as an integrable connection:

```
> C2:=OreModules[Connection](R2,Alg2);
```

$$C2 := \left[\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{1}{2}\,x1 \\ 0 & \frac{1}{2}\,d & 0 & x2 \\ 0 & 0 & 0 & \frac{(-3-d)\,x1}{-4\,x2+x1^2} \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{2}\,x1 \\ 0 & 0 & 0 & \frac{6+2\,d}{-4\,x2+x1^2} \end{bmatrix}\right] \tag{3}$$

If we want to know the basis of the associated module in which the connection is written, we can use the procedure KBasis of OreModules:

```
> OreModules[KBasis](R2,Alg2);
```

$$\left[\lambda_1, \lambda_1\,d2, d1\,\lambda_1, \lambda_1\,d2^2\right] \tag{4}$$

We define the right-hand sides of the connection.

```
> b1:=vector(4,[0,0,1,0]);
  b2:=vector(4,[0,0,0,0]);
```

$$b1 := \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$
$$b2 := \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

We use our procedures for computing rational solutions of the integrable connection. The right-hand sides are given as last input.

```
> RationalSolutions(C2,[x1,x2],['param',[d],'rhs',[b1,b2]]);
```

$$\left[\begin{bmatrix} \frac{1}{2}\,x1^2\,d+2\,x2 & x1 & 1 \\ 2 & 0 & 0 \\ x1\,d & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{2}\,x1^2 \\ 0 \\ x1 \\ 0 \end{bmatrix}\right] \tag{6}$$

The ouput contains a matrix whose columns form a basis of rational solutions of the homogeneous integrable connection and a vector that is a particular
solution of the non-homogeneous integrable connection. A solution is then given by the sum of a linear combination of the columns of the matrix and the vector.

BrycLetac system in dimension 3 - Gaussian Case

We define the OreAlgebra (needed for OreModules):

```
> Alg3:=DefineOreAlgebra(diff=[d1,x1],diff=[d2,x2],diff=[d3,x3],
  polynom=[x1,x2,x3],comm=[d]):
```

We give the equations of the system:

```
> R3 := matrix(3,1,[-d*d2+d1^2-x2*d2^2-2*x3*d2*d3,-(1/2)*d*d3+2*d1*
  d2+x1*d2^2-x3*d3^2,2*d1*d3+d2^2+2*x1*d2*d3+x2*d3^2]);
```

$$R3 := \begin{bmatrix} -d\, d2 + d1^2 - x2\, d2^2 - 2\, x3\, d2\, d3 \\ -\frac{1}{2}\, d\, d3 + 2\, d2\, d1 + x1\, d2^2 - x3\, d3^2 \\ 2\, d1\, d3 + d2^2 + 2\, x1\, d2\, d3 + x2\, d3^2 \end{bmatrix} \quad (7)$$

We write the system as an integrable connection:

```
> C3:=OreModules[Connection](R3,Alg3):
```

If we want to know the basis of the associated module in which the connection is written, we can use the procedure KBasis of OreModules:

```
> OreModules[KBasis](R3,Alg3);
```

$$\left[\lambda_1, \lambda_1\, d3, \lambda_1\, d2, \lambda_1\, d1, \lambda_1\, d3^2, \lambda_1\, d2\, d3, \lambda_1\, d1\, d3, d3^3\, \lambda_1\right] \quad (8)$$

We define the right-hand sides:

```
> b1:=vector(8,[0$3,1,0$4]);
  b2:=vector(8,[0$8]);
  b3:=eval(b2);
```

$$b1 := \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$b2 := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$b3 := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

We use our procedure for computing rational solutions of the integrable connection:

```
> RationalSolutions(C3,[x1,x2,x3],['rhs',[b1,b2,b3],'param',[d]]);
```

$$\left[\begin{bmatrix} \frac{1}{6}\, x1^3\, d^2 + x1\, d\, x2 + 4\, x3 & \frac{1}{2}\, x1^2\, d + x2 & x1 & 1 \\ 4 & 0 & 0 & 0 \\ x1\, d & 1 & 0 & 0 \\ \frac{1}{2}\, x1^2\, d^2 + d\, x2 & x1\, d & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{2}\, x1^2 \\ 0 \\ 0 \\ x1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right] \quad (10)$$

The ouput contains a matrix whose columns form a basis of rational solutions of the homogeneous integrable connection and a vector that is a particular
solution of the non-homogeneous integrable connection. A solution is then given by the sum of a linear combination of the columns of the matrix and the vector.