```
> restart:
```

We load our package:

```
> with(IntegrableConnections);
```

[Eigenring, Evalm, G_matrix, G_matrix0, HyperexponentialSolutions, Id, Mapply, MatrixColumnPrimpart, MatrixColumnPrimpartRat, Mexpsolde, Mpolsolde, Mratsolde, Msylvester, Mylinsolve, NNIexponents, PolynomialSolutions, RationalSolutions, Reduction, TestIntegrabilityConditions, Trigonalize, backsubs, candidates, col_pval, colechelon, colval, complement, convert_back, denomatrix, denomvector, direct_ratsol, eval_block_reduced, evalam, exp_parts, formal_reduce, gen_sylvester, getargs, good_form, intDiff, int_diff_split, l_colechelon, l_column_rank, l_moser_reduce, l_qtcd, l_super, l_super_reduce, lc_evala, lc_evalam, localize, log_collect, log_free, mat_block_reduce, mat_change_exp_part, mat_check, mat_coeff, mat_colvaluation, mat_convert, mat_copy, mat_difference, mat_eval, mat_get_dim, mat_get_exp_part, mat_get_ext, mat_get_indicial_polynomial, mat_get_inv_transformation, mat_get_order, mat_get_ramification, mat_get_terms, mat_get_transformation, mat_get_type, mat_lc, mat_lead_mon, mat_product, mat_pval, mat_subs, mat_sum, mat_swap, mat_to_simple, mat_tools, mat_transform, mat_val, mat_valuation, matrix_series, matval, minimalIntegerRoot, mult, my_linsolve, mydegree, myequaind, new_matrix, newton, newton_polygon, nonzero, normalm, pencil_block_reduce, pencil_solve, pgaussUnimod, poly_pval, poly_sols, poly_val, prank, pval, qtcd, ramified_case, ramified_reduction, randomMatrix, randomPoly, randomRat, ratsuper, reduceColRank4, reg_sols, regular_series, remove, rowechelon, seq_evala, sim_sylvester, simple_eval, simple_sols, sortColumns, super_form, system_expsolde, tensor_prod, transform, truncate, utils, val, val1, val_ldegree, valuation, vectdegree] **(1)**

The library linalg is also needed.

```
> with(linalg):
```

Consider the connection given by the two following matrices:

```
> C[1]:= Matrix(2, 2,[0,2/((x1+x2)*(-x2+x1)),-x1/(-x2+x1),-1/(x1+x2)]
  );
  C[2]:= Matrix(2, 2,[-2/(x1+x2),-2/((x1+x2)*(-x2+x1)), (x2^2+2*x1^2-
  x1*x2)/((x1+x2)*(-x2+x1)),  1/(x1+x2)]);
```

$$C_1 := \begin{bmatrix} 0 & \frac{2}{(x1+x2)\,(-x2+x1)} \\ -\frac{x1}{-x2+x1} & -\frac{1}{x1+x2} \end{bmatrix}$$

$$C_2 := \begin{bmatrix} -\frac{2}{x1+x2} & -\frac{2}{(x1+x2)\,(-x2+x1)} \\ \frac{x2^2+2\,x1^2-x1\,x2}{(x1+x2)\,(-x2+x1)} & \frac{1}{x1+x2} \end{bmatrix} \quad \textbf{(2)}$$

We use our procedure to compute the eigenring of the system:

```
> Eig:=Eigenring([C[1],C[2]],[x1,x2]);
```

$$Eig := \left[\begin{bmatrix} -\frac{2\,x1}{x1+x2} & -\frac{2}{x1+x2} \\ \frac{x1^2+x2^2}{x1+x2} & -\frac{2\,x2}{x1+x2} \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right] \quad \textbf{(3)}$$

The eigenring contains a non-trivial element given by the following matrix:

```
> P:=Eig[1];
```

$$P := \begin{bmatrix} -\frac{2\,x1}{x1+x2} & -\frac{2}{x1+x2} \\ \frac{x1^2+x2^2}{x1+x2} & -\frac{2\,x2}{x1+x2} \end{bmatrix} \quad \textbf{(4)}$$

We can thus use this matrix to decompose the system as follows. The matrices of the decomposed system are given by the diagonal matrices CC[1] and CC[2] below:

```
> jordan(P,'T'):
```

The matrix giving the change of variable is T:

```
> evalm(T);
```

$$\begin{bmatrix} -\frac{1}{2}\,\frac{-x1-x2+\mathrm{I}\,x1-\mathrm{I}\,x2}{x1+x2} & \frac{1}{2}\,\frac{x1+x2+\mathrm{I}\,x1-\mathrm{I}\,x2}{x1+x2} \\ \frac{\frac{1}{2}\,\mathrm{I}\,(x1^2+x2^2)}{x1+x2} & -\frac{\frac{1}{2}\,\mathrm{I}\,(x1^2+x2^2)}{x1+x2} \end{bmatrix} \quad \textbf{(5)}$$

we compute its inverse:

```
> invT:=inverse(T):
```

We perform the change of variable that decompose the connection. Note that, we have to work over Q(I) with I^2=-1 to perform this decomposition.

```
> CC[1]:=map(normal,evalm(inverse(T)&*(C[1]&*T-map(diff,T,x1))));
  CC[2]:=map(normal,evalm(inverse(T)&*(C[2]&*T-map(diff,T,x2))));
```

$$CC_1 := \begin{bmatrix} \frac{\mathrm{I}\,x1\,(x1+x2+\mathrm{I}\,x1-\mathrm{I}\,x2)}{(-x2+x1)\,(x1^2+x2^2)} & 0 \\ 0 & \frac{\mathrm{I}\,x1\,(-x1-x2+\mathrm{I}\,x1-\mathrm{I}\,x2)}{(-x2+x1)\,(x1^2+x2^2)} \end{bmatrix}$$

$$CC_2 := \quad \textbf{(6)}$$

$$\begin{bmatrix} -\frac{2\,\mathrm{I}\,x1^2+x1\,x2-\mathrm{I}\,x1\,x2-x2^2+\mathrm{I}\,x2^2}{(-x2+x1)\,(x1^2+x2^2)} & 0 \\ 0 & \frac{2\,\mathrm{I}\,x1^2-x1\,x2-\mathrm{I}\,x1\,x2+x2^2+\mathrm{I}\,x2^2}{(-x2+x1)\,(x1^2+x2^2)} \end{bmatrix}$$