A l'occasion du Doctorat Honoris Causa Décerné à Jonathan Borwein

par

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Jonathan M. Borwein, FRSC

Director, Centre for Experimental and Constructive Mathematics
Department of Mathematics and Statistics, Simon Fraser University
Burnaby, BC. V5A 1S6, Canada

E-mail: `jborwein@cecm.sfu.ca`

URL: `www.cecm.sfu.ca/personal/jborwein`

I intend to tell you a bit about myself, my research and the centre I work at. I decided, however, I should start with a disclaimer:

Three quotes from Michael Faraday:

> "A centre of excellence is, by definition, a place where second class people may perform first class work."
>
> "A truly popular lecture cannot teach, and a lecture that truly teaches cannot be popular."
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> "The most prominent requisite to a lecturer, though perhaps not really the most important, is a good delivery; for though to all true philosophers science and nature will have charms innumerably in every dress, yet I am sorry to say that the generality of mankind cannot accompany us one short hour unless the path is strewed with flowers."

- Excerpted from "Michael Faraday — and the Royal Institution, the genius of man and place", by J. M. Thomas, Adam Hilger, Bristol 1991.

I am both honoured and delighted to be standing here today to accept this *doctoris honoris causa* and to talk briefly about my professional life. I wish to begin by expressing my deep gratitude to the University of Limoges for conferring this honour on me. I especially wish to extend thanks to my old friend and colleague, Michel Thera, with whom I have interacted most stimulatingly for more that twenty years. As the citation indicates, it is particularly gratifying to have Limoges as the author of the degree. Indeed, Limoges is signal in Europe in having internationally recognized researcher groups in all the major areas in which I have worked: theoretic and computational optimization, analytic and computational number theory, symbolic and numeric computation.

I have been fortunate to be born into an academic family. Indeed both my parents, are here in Limoges to share in the proceedings. Dr. Bessie — an anatomist, administrator and life-long activist — and my mathematician father David Borwein, FRSE, head of Pure Mathematics at University of Western Ontario from 1967 to 1989, President of the Canadian Mathematical Society (1984–1986) and my frequent co-author. We have eleven joint papers, ten of which followed my father's somewhat mis-described "retirement", and are still counting. His Ph.D. supervisor was L.S. Bosanquet whose own supervisor was G.H. Hardy. Hardy was the leading English mathematician of the first half of this century; he is most famous perhaps for "discovering " the great self-educated Indian mathematician Srinivasa Ramanujan (1887–1920) some fragment of whose work I have been privileged with my brother Peter to explore, expose and extend. Indirectly we inherited a taste for classical analysis from Hardy.

Both my brother and I ultimately became academic mathematicians and not surprisingly have from time to time mulled over what factors lead us to take up the same vocation. I started University determined to be a historian. Neither of us was in any sense "hot-housed". In my undergraduate career I had precisely one lecture from my father; otherwise he assiduously scheduled classes so as to avoid our meeting. The only exception being a 1957 bet with his colleagues in St. Andrews, Scotland — for a large quantity of of cheese — that he could teach his six year old son to solve two-by-two simultaneous linear equations by making it into a game. In still recently post-war Britain I was so taught and while comprehending neither reason nor rationale I loved playing this mysterious game and taught my best friend also to play.

From David and Bessie I learned of the quiet satisfactions of an intellectual life; but not of a life lived in a vacuum. I was taught to make yet one more revision to a paper and to savour the polish and finish it provided. And finally, my overarching memory is of my father, at frequent parties arranged by my more outgoing mother, playing generously if not exuberantly the role of host. By mid-evening his eyes would slightly glaze and a stream of cocktail napkins would issue forth covered with formulae and expressions in his careful and concise italic script.

I have been unusually privileged mathematically. I have worked intensively with both brother and father as equals and have known my father as an intellectual peer for more than a quarter century.

I originally went to Oxford to study pure functional analysis but quickly moved to more applied optimization topics — and am very grateful I did since it has allowed me to move between fields and work with a variety of wonderful collaborators. Above all it prepared me to work in areas with which I was only passingly familiar and to take intellectual risks.

As a graduate student in Oxford I was entranced by the distinguished (mathematical and linguistic) philosopher Michael Dummett who taught me about Frege and Heyting, expected me to prove the "Independence of the Continuum Hypothesis" in viva, and lectured seemingly sans notes with a lucidity that provided "camera ready copy". His curiously two-tone hair (bright yellow at the front, silver gray at the back) was explained by my sudden realization half way through a course that each cycle across the proscenium had regular nicotine producing features. Inhale from the cigarette in the elegant ivory holder; exhale, expostulate and excogitate, run fingers through yellow-gray mane; repeat for duration of hour.

And Brian Birch the (then and now) leading British number theorist whose elliptic slang left me temporarily reeling. Phrases something like "kill the integral and shove the function to the left" I eventually decoded as a good prescription for contour integration. Equally, "the zeta function associated with an elliptic curve is exactly what you thought it would be" proved more useful with hindsight than on first meeting.

But in fact this rich stew of pure, applied and philosophical mathematics has matured over the years and allowed me build the *Centre for Experimental and Constructive Mathematics* to which I will return later.

My thesis research — as should be the case — has provided questions to which my students my co-workers and I have returned over the years. Just this last spring, my most recent Doctoral student (Wang Xianfu) and I answered a question I had first posed in the seventies: most non-expansive functions are so badly behaved that they oscillate as much as they can at every point.

I returned from Oxford to Dalhousie University in 1974 and came under the wing of Michael Edelstein at Dalhousie, an intense Polish-Israeli functional analyst who by example and engagement converted a generation of young analysts into full blooded independent researchers. He provided much of the functional analytic training I had avoided in Oxford while playing bridge for the University.

I moved to Carnegie-Mellon in the early eighties and I was lucky enough to work with many exciting operations researchers and mathematical economists including Dick Duffin, a distinguished mathematician and engineer. Dick, who had trained John Nash (a truly extraordinary man and a future Economics laureate), Raul Bott (himself the supervisor of future Fields medalists) and Hans Weinberger among others, was a man absolutely without pretensions who forcibly taught me that great depth of insight and what Louis Wolpert has nicely termed “a passion for science” had no need of flashy or pretentious packages.

The problems of teaching large classes of management science and business students (at Carnegie-Mellon and after returning to Dalhousie in 1982) provided the impetus for my first involvement with computer assisted mathematics. We wrote very simple software which could assist students in solving *linear programmes*, then and now, one of the work-horses of the algorithmic world; and which are horrifyingly unpleasant to execute at the blackboard. Our ‘idiot-pivoter’ was a huge success and spread throughout the university. The pivoter answered a need and I was hooked on intelligent use of technology.

Of course, 1982 was also close to the birth of the true personal computer and I bought my first ones at that time. They were liberating especially because they allowed academics to learn the computing they had failed to master on punch cards and tape in the privacy of their offices — with no one to record their mistakes. In 1983 we bought ’Lisa’s’. These machines were the first true workstations. Though they proved an evolutionary dead end when Apple brought out the Macintosh and killed the technically superior Lisa for marketing reasons they were wonderful. In our ignorance of best practice we computed certain *mathematical invariants* that were viewed as intractable.

In 1982 my brother and I — who had ended up living in the same apartment house — decided to study Pi together. At age thirty, we had never discussed mathematics, and thought it would be nice to write a little paper together on Pi, the area of a unit circle and how to compute it quickly and other elementary constants quickly. Over the years, that little paper has generated many joint publications, an article in Scientific American (activated at www.cecm.sfu.ca/organics/papers/borwein/index.html) several shared prizes and four joint books. It introduced us to wildly diverse people: some cranks and hobbyists but also including the great Indian astro-physicist Chandrasekhar, Freeman Dyson of the Institute for Advanced Study, John Todd and Olga Tausky from Cal Tech and the extraordinary Russian Chudnovsky brothers. Moreover, it gave me a topic about which I could and frequently do lecture fruitfully to non-mathematicians, and talk about to the press.

In 1982, two million digits of Pi had been computed. Now sixty seven billion have. A generation ago this was considered impossible in the rest of time. But that is another story. I do wish to note that the algorithm used in many of these computations is two lines long (two very good lines — easy to code and hard to explain).

(*A fourth-order algorithm.*) Set $a_0 = 6 - 4\sqrt{2}$ and $y_0 = \sqrt{2} - 1$. Iterate

$$\boxed{y_{k+1} = \frac{1 - (1 - y_k^4)^{1/4}}{1 + (1 - y_k^4)^{1/4}}} \tag{1}$$

$$\boxed{a_{k+1} = a_k(1 + y_{k+1})^4 - 2^{2k+3} y_{k+1}(1 + y_{k+1} + y_{k+1}^2)}$$

Then a_k converges *quartically* to $1/\pi$: nineteen *full precision* steps gives 100 billion digits.

I discovered this algorithm on an 8K portable Radio-Shack computer in 1985 in the back of a car. In Richard Dawkin’s terms it is a successful “meme”, (“Memes are the basic building blocks of our minds and culture, in the same way that genes are the basic building blocks of biological life.”) and it gives me great pleasure to watch it replicate around the world wide web and elsewhere.

At John Todd's suggestion we had begun to explore Ramanujan's work on Pi. Since he worked largely intuitively and without proof we began to heavily use an early version of Maple so that we could check his assertions before we tried to prove them or digest them. Maple is a computer algebra system — a subject about which Dominique Duval and her colleagues in Limoges are very knowledgeable. These systems are now very powerful. In the process we became early exponents of a growing field called 'Experimental Mathematics'.

We have had some striking successes. One I call *'Pentium farming' for binary digits.* Bailey, P. Borwein and Plouffe (1996) discovered a series for π which allows one to compute hexadecimal digits of π *without* computing prior digits. This is totally counter-intuitive. The key, found by computer search is:

$$\boxed{\pi = \sum_{k=0}^{\infty} \left(\frac{1}{16}\right)^k \left(\frac{4}{8k+1} - \frac{2}{8k+4}\frac{1}{8k+5} - \frac{1}{8k+6}\right)}. \tag{2}$$

Knowing an algorithm would follow they spent several months hunting for such a formula. This is a form of *reverse mathematical engineering.* Once found, easy to prove in Mathematica, Maple or by hand.

In August 1998 Colin Percival (a 17 year old student at Simon Fraser) finished what is now called an "embarassingly parallel" computation of *five trillionth bit* (using 25 machines over the world wide web). In *Hexadecimal*:

$$\underline{0}7E45733CC790B5B5979$$

Similarly the binary digits of π starting at the 40 *trillionth bit* are

$$\underline{0}0000111110011111.$$

During a 1996 study of *Riemann's zeta function* David Bradley and I used similar search methods to discover a marvelous finite sum:

$$\boxed{\sum_{k=1}^{n} \frac{2n^2}{k^2}\,\frac{\prod_{i=1}^{n-1}(4k^4+i^4)}{\prod_{i=1\ i\neq k}^{n}(k^4-i^4)} = \binom{2n}{n}} \tag{3}$$

of a quite unusual form, after making a lucky mistake in an electronic Petrie dish. This identity was recently proved by Almkvist and Granville perhaps shedding light on the irrationality of $\zeta(7)$?. Recall that $\zeta(2N+1)$ is not proven irrational for $N > 1$.

The Hungarian mathematician Paul Erdös, when shown (3) shortly before his death, rushed off. Twenty minutes later he returned saying he did not know how to prove it but if proven it would have implications for Apéry's famous 1976 result that '$\zeta(3)$ is irrational'. Paul, whom I first met when I was a little boy (an 'epsilon' was is term) and he stayed with my family in Scotland, has received an honorary degree from the university of Limoges.

Three quotes from David Berlinski:

> "Mathematicians are like pilots who maneuver their great lumbering planes into the sky without ever asking how the damn things stay aloft."

> "The computer has in turn changed the very nature of mathematical experience, suggesting for the first time that mathematics, like physics, may yet become an empirical discipline, a place where things are discovered because they are seen."
>
> "The existence and nature of mathematics is a more compelling and far deeper problem than any of the problems raised by mathematics itself."

- From a review of "The Pleasures of Counting" by T. W. Korner (Cambridge 1996) in *The Sciences*, July/August 1997, pp. 37–41).

Since 1992, I have overseen the *Centre for Experimental and Constructive Mathematics* with help from many others notably my brother. The Centre was founded on the premise that there is a massive change taking place in mathematics. In 1993 my brother and I wrote in the *Notices of the American Mathematical Society*:

> **Preamble:** Over the last quarter Century and especially during the last decade, a dramatic 're- experimentalization' of Mathematics has begun to take place. In this process, fueled by advances in hardware, software and theory, the computer plays a laboratory role for pure and applied mathematicians; a role which in the eighteenth and nineteenth centuries the physical sciences played much more fully than in our century.
>
> Operations previously viewed as non-algorithmic, such as indefinite integration, may now be performed within powerful symbolic manipulation packages like Maple, Mathematica, Macsyma, and Scratchpad to name a few. Similarly, calculations previously viewed as "practically" non-algorithmic or certainly not worth the effort, such as large symbolic Taylor expansions are computable with very little programming effort.
>
> New subjects such as computational geometry, fractal geometry, turbulence, and chaotic dynamical systems have sprung up. Indeed many second-order phenomena only become apparent after considerable computational experimentation. Classical subjects like number theory, group theory and logic have received new infusions. The boundaries between mathematical physics, knot theory, topology and other pure mathematical disciplines are more blurred than in many generations. Computer assisted proofs of "big" theorems are more and more common: witness the 1976 proof of the Four Colour theorem and the more recent 1989 proof of the non-existence of a projective plane of order ten (by Clement Lam et al at Concordia).
>
> There is also a cascading profusion of sophisticated computational and graphical tools. Many mathematicians use them but there are still many who do not. More importantly, expertise is highly focused: researchers in partial differential equations may be at home with numerical finite element packages, or with the NAG or IMSL Software Libraries, but may have little experience with symbolic or graphic languages. Similarly, optimizers may be at home with non-linear programming packages or with Matlab. The learning curve for many of these tools is very steep and researchers and students tend to stay with outdated but familiar resources long after these have been superseded by newer software. Also, there is very little methodology for the use of the computer as a general adjunct to research rather than as a means of solving highly particular problems. "

All of this is truer today than in 1993. We have been most interested in building tools for real-time research collaboration at a distance. We have tried with some success to perform real mathematical experiments. We are most centrally concerned with using the computer as an aid to insight. Picasso once said. "Computers are useless, they can only give you answers." He was wrong. We espouse a humanist philosophy much as enunciated by Reuben Hersh in the *American Mathematical Monthly* August-September 1995, 589–594.

1. **Mathematics is human**. It's part of and fits into human culture. (Not Frege's abstract, timeless, tenseless, objective reality.)
2. **Mathematical knowledge is fallible**. Like science, mathematics can advance by making mistakes and then correcting and recorrecting them. (This "fallibilism" is brilliantly argued in Lakatos' Proofs and Refutations.)
3. **There are different versions of proof or rigor,** depending on time, place, and other things. The use of computers in proofs is a nontraditional version of rigor.
4. **Empirical evidence, numerical experimentation, probabilistic proof** all help us decide what to believe in mathematics. Aristotelian logic isn't necessarily always the best way of deciding.
5. **Mathematical objects are a special variety of social-cultural-historical object**. We can tell mathematics from literature or religion. Nevertheless, mathematical objects are shared ideas, like Moby Dick in literature, or the Immaculate Conception in religion.

I have also maintained a deep interest in less algorithmic mathematics, although it is my experience that the computer can be profitably used through pure and applied science. In the mid eighties I was able with David Preiss to prove the first "smooth variational principle". This is a result which allows mathematicians to apply classical — and enormously successful — techniques from Newtonian calculus in situations where the objects involved are highly non-differentiable and traditionally viewed as quite unruly. I spoke last week in Italy on some newer related work. Despite its abstract nature much of my insight came from my ability to draw subtle pictures quickly on my computer and to develop reliable insight.

I view mathematics as the *language of high technology.* Folks around my Centre are more or less deeply involved in web-based computation, high-performance networking, distance learning, human-computer interface design, building interactive dictionaries and 'living books', medical imaging, financial mathematics, disaster mediation, the manned Mars mission and many other things.

The work on interactive dictionaries started in a farm house outside of Limoges in 1985, and has lead to the creation of a modestly successfully educational software company. We also have gone out of our way to make liaisons with other scientists, educators and students, businesses, the media and others. If we don't do a good job of "selling" mathematics, why should the public "buy" it?

Mathematics is, with justification, a very conservative discipline. Most mathematicians still espouse an idealist philosophy of mathematics and so 'Experimental Mathematics' remains quite controversial. Thus, I will finish with another 'expert-witness'.

Thomas Kuhn

> "The issue of paradigm choice can never be unequivocally settled by logic and experiment alone $\cdots$ in these matters neither proof nor error is at issue. The transfer of allegiance from paradigm to paradigm is a conversion experience that cannot be forced."
>
> - The author of *The Structure of Scientific Revolutions* quoted in Ed Regis's, *Who got Einstein's Office?* Addison-Wesley, 1986.

That said, the world of science is changing very rapidly. As Gary Taubes captures in Science, writing about particle physics (at the cutting-edge of these changes) :

> "As a result, no one theorist or even a collaboration does definitive work. Instead, the field progresses like a jazz performance: A few theorists develop a theme, which others quickly take up and elaborate."

Above all mathematics in the presence of computers can be enormous fun and *homo sapiens* is also *homo ludens.* I hope I have been able to give you some sense of the work that animates me and many of my collaborators. Once again, thank you for doing me this great honour.

Jonathan Borwein
Simon Fraser University
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