On a conjecture concerning minus parts in the style of Gross
Radan Kučera, Masaryk University, Brno, Czech Republic
(A joint work with Cornelius Greither,
Universität der Bundeswehr München, Germany)

In the eighties, B. Gross introduced a conjecture which is close to Stark’s
conjectures inasmuch as it postulates a link between L-values and regulators,
but which differs from Stark’s conjectures and already from Dirichlet’s class
number formula in a very important aspect: the regulators are not complex
or $p$-adic numbers, arising as determinants of logarithms of certain algebraic
numbers, but they lie in an appropriate quotient of the augmentation filtra-
tion of $\mathbb{Z}[G]$, where $G$ is the Galois group of the abelian field extension $K/F$
under consideration, and they are obtained as determinants of matrices made
from certain local Artin symbols.

Recently D. Burns discovered a rather general conjectural framework
welding together Stark-type and Gross-type conjectures. For a real abelian
extension $K/\mathbb{Q}$ this applies in two steps. First there is a Stark unit $\eta_K$ (whose
existence is proven in this case, not just a conjecture), and then there is a
statement concerning the “position of $\eta_K$ within the whole group $\mathcal{O}_K^*$” in
terms of a Gross regulator. At the first stage, a classical regulator is used,
namely a determinant involving the logarithms of the conjugates of $\eta_K$. At
the second stage, the Gross-style regulator is used to obtain a conjectural
congruence in $\mathbb{Z}[G]$ modulo a high power of the augmentation ideal. (The
whole setup is generalized to any finite abelian extension of global fields, a
real abelian extension of $\mathbb{Q}$ was considered here just to keep things as simple
as possible.)

In subsequent work of A. Hayward, where Burns’s conjectures are dis-
cussed and in some cases proved, another conjecture comes into play which
may be considered as the “minus part” of Burns’s conjecture for extensions
$K/F$ where $F$ is an imaginary quadratic field and $K$ is absolutely abelian.
This “minus conjecture” equates, up to constant factors, the leading term
of a Stickelberger element and a regulator made up from $S$-units in the mi-
nus part. (In fact, this “minus conjecture” is a very special case of what is
called “the conjecture of Gross on tori”, on which nothing much seems to be
known.) In particular, the “minus conjecture” gives what should be obtain-
able, roughly speaking, by dividing a conjectural equation for $K/F$ by the
corresponding equation for $K^+/\mathbb{Q}$, where $K^+$ is the maximal real subfield of
$K$. However, this division process often does not make sense (all quantities
involved may be zero). So we are interested in direct proofs of the “minus
conjecture” which do not use the conjectural equations for either $K/F$ or
$K^+/\mathbb{Q}$. This talk presents some (very partial) results in this direction.