Growth of Selmer groups in dihedral extensions

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In joint work with Barry Mazur, we obtain lower bounds for Selmer ranks of elliptic curves over dihedral extensions of number fields.

If F/k is a dihedral extension of number fields of degree $2n$ with n odd, and E is an elliptic curve over k that has odd rank over the quadratic extension K of k in F, then standard conjectures (and a root number calculation) predict that $E(F)$ has rank at least n. The only case where one can presently prove anything close to this bound is when K is imaginary quadratic, and $E(F)$ contains Heegner points.

Mazur and I prove unconditionally that if n is a power of an odd prime p, F/K is unramified at all primes where E has bad reduction, all primes above p split in K/k, and the p-Selmer corank of E/K is odd, then the p-Selmer corank of E/F is at least n. This provides a large class of examples of $\mathbf{Z}_p^d$-extensions where the Selmer module is not a torsion Iwasawa module.