On a conjecture concerning minus parts in the style of Gross

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In the eighties, B. Gross introduced a conjecture which is close to Stark's conjectures inasmuch as it postulates a link between L-values and regulators, but which differs from Stark's conjectures and already from Dirichlet's class number formula in a very important aspect: the regulators are not complex or p-adic numbers, arising as determinants of logarithms of certain algebraic numbers, but they lie in an appropriate quotient of the augmentation filtration of $\mathbb{Z}[G]$, where G is the Galois group of the abelian field extension K/F under consideration, and they are obtained as determinants of matrices made from certain local Artin symbols.

Recently D. Burns discovered a rather general conjectural framework welding together Stark-type and Gross-type conjectures. For a real abelian extension $K/\mathbb{Q}$ this applies in two steps. First there is a Stark unit η_K (whose existence is proven in this case, not just a conjecture), and then there is a statement concerning the "position of η_K within the whole group $\mathcal{O}_K^*$" in terms of a Gross regulator. At the first stage, a classical regulator is used, namely a determinant involving the logarithms of the conjugates of η_K. At the second stage, the Gross-style regulator is used to obtain a conjectural congruence in $\mathbb{Z}[G]$ modulo a high power of the augmentation ideal. (The whole setup is generalized to any finite abelian extension of global fields, a real abelian extension of $\mathbb{Q}$ was considered here just to keep things as simple as possible.)

In subsequent work of A. Hayward, where Burns's conjectures are discussed and in some cases proved, another conjecture comes into play which may be considered as the "minus part" of Burns's conjecture for extensions K/F where F is an imaginary quadratic field and K is absolutely abelian. This "minus conjecture" equates, up to constant factors, the leading term of a Stickelberger element and a regulator made up from S-units in the minus part. (In fact, this "minus conjecture" is a very special case of what is called "the conjecture of Gross on tori", on which nothing much seems to be known.) In particular, the "minus conjecture" gives what should be obtainable, roughly speaking, by dividing a conjectural equation for K/F by the corresponding equation for $K^+/\mathbb{Q}$, where K^+ is the maximal real subfield of K. However, this division process often does not make sense (all quantities involved may be zero). So we are interested in direct proofs of the "minus conjecture" which do not use the conjectural equations for either K/F or $K^+/\mathbb{Q}$. This talk presents some (very partial) results in this direction.